Demographic shift, consumer capital, and productivity dispersion

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May 2024

Abstract

This paper proposes a link between demographic shift and increasing heterogeneity across firms regarding revenue productivity. From data on public US firms, I find evidence of demand side effects, whereby firms in sectors whose outputs are consumed more by older households have more dispersed R&D investment and revenue productivity. Using a quantitative model, I show that when older consumers have stronger consumption habits, a rise in the share of older consumers in the economy could lead to larger divergence across firms. Through the lens of the model, the increase in the share of older consumers over the 1990-2019 period could explain half of the observed increase in revenue productivity dispersion.

1 Introduction

Over the past 40 years, there has been a steady rise in revenue productivity dispersion across firms. The standard deviation of log revenue productivity across US public firms, after accounting for industry-time fixed effects, rose by 50%, from around 0.2 in 1980 to around 0.3 in 2020. Productivity is closely related to other firm level outcomes, as such this increase in revenue productivity dispersion has been accompanied by higher dispersion in firm size - hence increasing market concentration; higher dispersion in wages - hence increasing labour income inequality; and higher dispersion in markups.

Understanding the source of this increase in productivity dispersion is also important. One candidate explanation is higher variance of exogenous shocks to firm productivity, or higher firm risk, a hypothesis proposed to explain higher revenue dispersion, as well as dispersion in other firm level outcomes, in times of recession (Bloom et al. (2018)). More varying shocks also implies depressed investment, from increased firm inaction (Bloom et al. (2018)) or higher credit constraints (Christiano et al. (2014)). Normal times today could be as risky as recessions 30 years ago, and investment in normal times today could be as depressed as in recessions 30 years ago.

But productivity is also shaped by endogenous firm decisions, especially over a long time period. In this paper, I propose an explanation for the rise in revenue productivity dispersion, via firms' endogenous response to the demographic shift, whereby the population becomes older on average. Leveraging previous studies on the importance of consumption habits in shaping demand, I focus on the effects associated with changing consumer demand due to the demographic shift. The broad idea is as follows. Young consumers as a source of consumer capital affect firms differently than older consumers as a source of consumer capital. The demographic shift implies a greater presence of capital stemming from older consumers, which in turn affects firm innovation incentives. The key point is that this affects firms differently: it increases innovation of leading firms relative to lagging firms. This then leads to higher dispersion in productivity, revenue productivity, and other firm level outcomes.

I document how the age composition of demand in an industry comoves with the divergence in R&D investments and revenue productivity across firms in that industry. I employ panel data on US industries from 1990 to 2019 that has measures of industry R&D spending difference between more productive firms and less productive firms, industry revenue productivity dispersion, and share of industry output consumed by older households. I find that when the consumption share of older households increase in an industry, that industry also experience a larger divergence in R&D spending among more productive firms and less productive firms, along with an increase in revenue productivity dispersion. As the population ages and the consumption share of older households increases in the economy, we might expect revenue productivity dispersion to increase as well.

I then build a model to study the mechanism of how the changing age composition of demand affect firms' R&D investments and productivity dispersion, and to quantify the effects. I start with the step-by-step innovation framework (Aghion et al. (2001)), where two firms compete in each industry, while investing in R&D to increase productivity to gain the upper hand on their competitor. Market leaders and followers arise as a result of this R&D race, with the leader being more productive, producing more goods, having higher profits, and charging higher markups. I augment this framework by allowing firms to build up consumer habits for their products (Ravn et al. (2006)). These habits are a form of consumer capital: they increase demand for the firm's product, and are generated from household consumption in previous periods. The addition of habits increases the advantage of being a leader: the more productive leader produces more, hence builds up more consumer capital, which in turn gives the leader a higher boost in increased demand.

A key assumption for the effect of demographic shift is that habits are only generated from the consumption of older households. This assumption is motivated by studies of demand patterns across age, whereby older consumers are more likely to stay with the same products (Bornstein (2021)), or have had the time to build up consumption habits for the products (Bronnenberg et al. (2012)). The fraction of older households in the economy governs the importance of consumer capital for firms. The demographic shift increases the share of older consumers, which gives leaders a larger boost in demand. This generates stronger incentives for leaders to innovate, compared to followers.

The model also reveals an opposing effect. Higher consumer capital also makes demand for the firm's product less elastic. This decreases the incentive for the leader to innovate, in favor of charging higher markups. So whether productivity dispersion increase or decrease depends on which effect dominates.

I calibrate the model to match the US in the 1970s. I then compare the resulting economy to one with a higher share of older households, at the level seen in the 2010s. I find an increase in revenue productivity dispersion, of around 50% of the increase observed in the data. In the model, aggregate markups and market concentration also increase, inline with documented trends in the data.

Turning to the question of whether the variance of shocks to firm productivity has increased, the model contains insight on endogenous objects the affects the measured variance of shocks in the data. The variance of shocks that is commonly backed out is proportional to the conditional variance of firm revenue productivity. This conditional variance is in part determined by firms' innovation decisions and productivity differences. Running regressions motivated by the model, I find no evidence of exogenous increases in the variance of shocks in the data.

Related Literature.

This paper is related to a recent strand of literature that explores causes for the increasing divergence in firm level outcomes. Apart from demographic shift, proposed explanations include lower rate of knowledge diffusion (Akcigit and Ates (2023)), lower interest rate (Liu et al. (2022)), heterogeneity in efficiency with respect to adoption of intangibles (De Ridder (2024)), and changing market structure and competitiveness (De Loecker et al. (2021)).

Other papers have studied the effect of demographic shift on firms. On the supply side, Hopenhayn et al. (2022) and Karahan et al. (2019) considers the effect of lower labour force growth rate, leading to a lower rate of new firm formation and older firms in the economy on average. Peters and Walsh (2021) furthers this argument in the the context of growth, where the decline in new firm formation results in lower creative destruction and innovation, leading to a decline in the aggregate growth rate. On the demand side, like this paper, Bornstein (2021) considers the affect of demographic shift through the channel consumer capital. Whereas the main focus of Bornstein (2021) is firm pricing decisions, this paper emphasizes the effect on firm technology investment incentives.

The model in this paper augments the step-by-step innovation framework (Aghion et al. (2001)) with consumer capital in the form of habits. A vast literature in industrial organization have studied how consumer capital, arising through lock-ins or switching costs, affects market competitiveness. For example, Beggs and Klemperer (1992) shows how lock-ins result in decreased competitiveness, increased price markups and profits; Dube et al. (2009) shows that switching costs could instead increase competitiveness, depending on the size of the cost. Bronnenberg et al. (2012) use consumer spending and migration data to estimate the strength and persistence of the effect that consumption habits have on product choice. In the macroeconomic context, Ravn et al. (2006) studies how the introduction of consumption habits affect firm cyclical behaviours in a dynamic stochastic general equilibrium model. Gourio and Rudanko (2014) studies consumer capital that arises from frictional matching between consumers and producers, and its effect on firm investment, sales, markups, and responsiveness to shocks. This paper contributes by exploring how consumer capital additionally affect firm productivity decisions. Moreover, my modeling approach allows me to leverage micro estimates on the strength of consumption habits to discipline the quantitative exercise.

2 Empirical movements in productivity dispersion and consumption share of older households

I first set up the step-by-step innovation framework as a guide for the empirical analysis. I then document the increase in divergence of firm productivity, along with the increase in divergence of R&D investment implied by the framework. Using variations within industries over time, I find evidence that when the consumption share of older households is higher in an industry, there is larger divergence in R&D investment and productivity for that industry.

2.1 Through the lens of the step-by-step innovation framework

Consider an industry with two firms that differ in productivity[1]. In each period t, any firm $i \in \{1, 2\}$ can invest in R&D to increase its next period productivity probabilistically. To achieve probability ι_{it} of a productivity increase, the firm has to spend $f(\iota_{it})$ on R&D, with $f(0) = 0, f' > 0, f'' \geq 0$. If successful, productivity increases proportionally by a factor $\lambda > 1$, so that $q_{it+1} = \lambda q_{it}$.

Productivity dispersion increases if the more productive firm targets a higher success probability. Suppose in period t, firm 1 has higher productivity than firm 2. Log productivity dispersion in the industry at time t is then proportional to $\log q_{1t} - \log q_{2t}$. The expected change in dispersion, from period t to period $t+1$, is proportional to

$$\mathbb{E}\left[\log q_{1t+1} - \log q_{2t+1}\right] - \left[\log q_{1t} - \log q_{2t}\right] = \left(\iota_{1t} - \iota_{2t}\right) \log \lambda, \tag{1}$$

which is positive if the innovation gap, $\iota_{1t} - \iota_{2t}$, is positive. Moreover, the change in dispersion is increasing in the innovation gap.

There are two relationships concerning the consumption share of older households that can be examined in the data. In the model below, consumer capital affects productivity dispersion through its effect on the innovation gap. So the first relationship is between the consumption share of older households and the innovation gap within an industry. The second, from equation (1), is between the consumption share of older households and the change in dispersion within an industry.

2.2 Rising revenue productivity dispersion and innovation gap

Before going to the comovements with the consumption share of older households, I examine the increase in revenue productivity dispersion, along with the increase in the innovation gap.

[1] These two firms can also be thought of as two groups of firms, a high productivity group and a low productivity group.

For revenue productivity, my preferred measure takes into account labour inputs as well as capital inputs. I estimate firm revenue productivity via production function estimation, using the sample of public US firms from 1980 to 2021, following the method in Flynn et al. (2019). I specify firms' production function as a flexible translog in capital and inputs, and allow the coefficients to vary with time and 2 digit industries. Details are provided in the appendix. As a comparison, I also consider log revenue per worker. While more straightforward and simple, this measure does not account for differences in capital across firms.

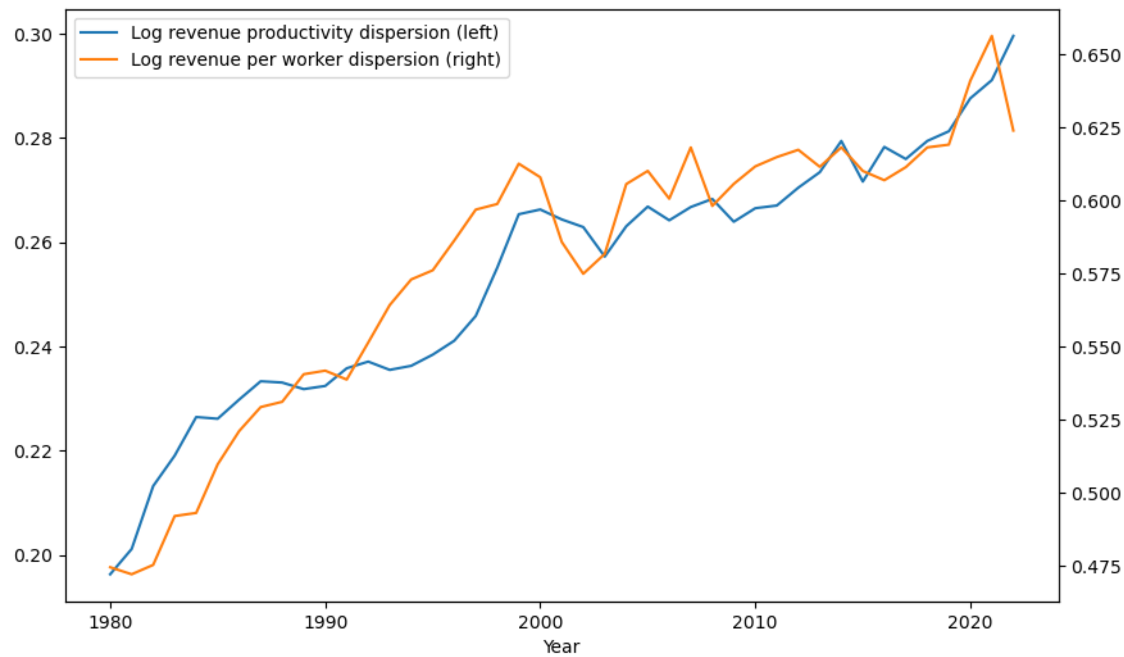

Figure 1: Revenue productivity dispersion

Aggregate revenue productivity dispersion have been increasing steadily overtime, as seen in Figure 1. From 1980 to 2020, revenue productivity dispersion increased by around 50%, from 0.2 to 0.3. This aggregate dispersion is calculated as the standard deviation of log revenue productivity across all firms, after controlling for 2 digit NAICS industry-time fixed effects. The trend in revenue productivity dispersion is also closely matched by log revenue per worker dispersion.

While revenue productivity is not a perfect measure of quality adjusted productivity, trends in the innovation gap could inform us if quality adjusted productivity dispersion have increased. Under equation (1), a widening innovation gap over time would hint at increased dispersion in underlying productivity. To obtain a measure of the innovation gap, I need a proxy for innovation probabilities, along with appropriate groupings of firms to take differences. For the former I can use R&D spending, although doing so require an assumption on the cost function f. I consider two cases, the first with f linear, hence using R&D spending directly to measure the gap; and the second with f exponential, hence using $\log(\text{R\&D} + 1)$ to measure the gap. To alleviate concerns of scale effects, I also consider the cases of using R&D spending per worker in place of R&D spending.

For firm groups, I divide firms into 3 bins based on their revenue productivity: (1) those below the 50^{th} quantile in their 4 digit industry-year, (2) those between the 50^{th} and 75^{th} quantile, and (3) those above the 75^{th} quantile. Then, for a proxy of innovation probabilities x, I calculate the mean of x, $\bar{x}_j$, for firms in each bin j, after taking out industry-year fixed effects. Taking the differences $\bar{x}_2 - \bar{x}_1$ and $\bar{x}_3 - \bar{x}_1$ yield the innovation gaps at different levels of productivity.

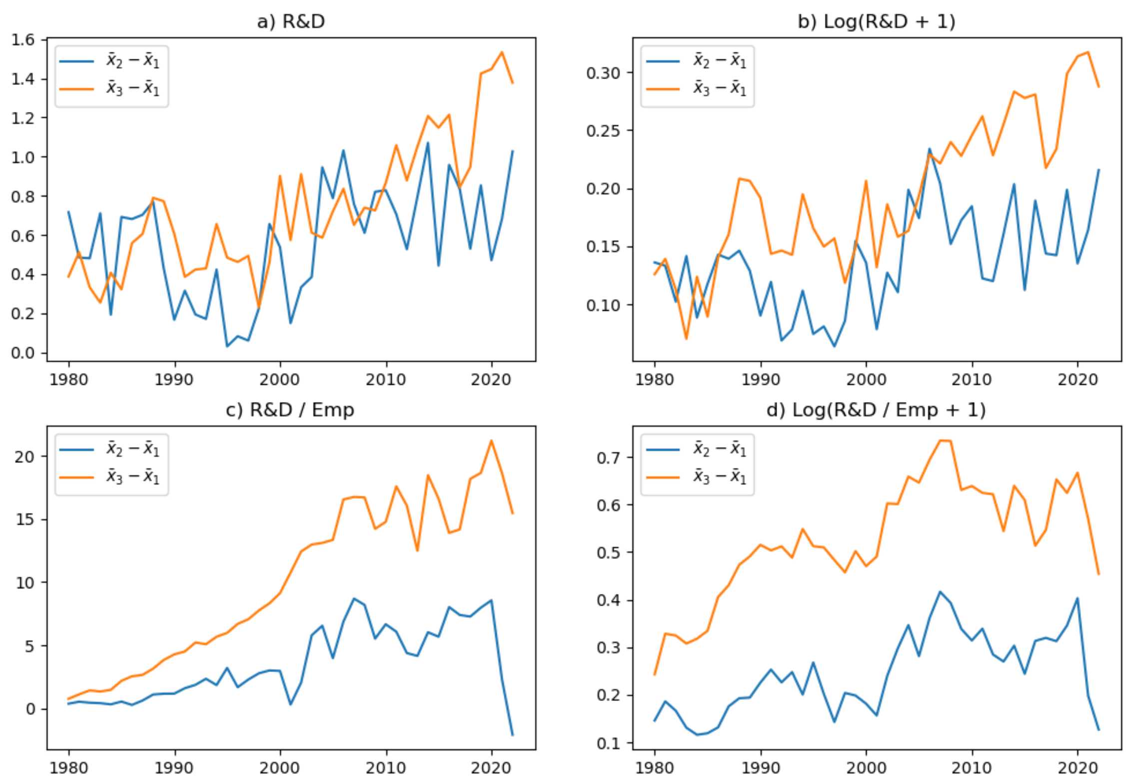

Figure 2: R&D spending differences across firm quantiles by revenue productivity

R&D spending for firms above the 50^{th} quantile have increasingly outpaced spending of firms below the 50^{th} quantile, as shown in Figure 2. The innovation gap has widened across the four measures considered. Moreover, this widening is larger for firms that are most productive, with $\bar{x}_3 - \bar{x}_1$ increasing more than $\bar{x}_2 - \bar{x}_1$.

2.3 Comovements with consumption share of older households

I next examine how the consumption share of older households comoves with the innovation gap and productivity dispersion within an industry.

I construct a panel data of industries from 1990 to 2019, with data on the share of expenditures by older households, measures of the innovation gap, and revenue productivity dispersion for each industry. For the expenditure share, I use the Consumer Expenditure Survey (CEX), crosswalking CEX industries to NAICS industries. Following Bornstein (2021), I choose 35 as the age cutoff for older households[2]. The measures of the innovation gap are constructed similar to subsection 2.2: I divide firms into bins based on their revenue productivity, then, for a proxy of innovation probabilities x, I calculate the mean of x, $\bar{x}_j$, for firms in each bin j. Motivated by Figure 2 where the more productive firm group widens their innovation gap more, I consider an additional bin: (4) firms above the 90^{th} quantile in their industry-year. This yields 3 innovation gaps for each proxy, in ascending level of productivity: $\bar{x}_2 - \bar{x}_1$, $\bar{x}_3 - \bar{x}_1$ and $\bar{x}_4 - \bar{x}_1$. Finally, for revenue productivity dispersion, I use the standard deviation of revenue productivity of firms within the industry. I define industries by their 3 digit NAICS codes, as various 4 digit NAICS industries in Compustat have few firms, leading to potentially large errors in revenue productivity dispersion.

The consumer demographic mechanism mainly affects consumer goods. To focus on such industries, I restrict my panel to industries that produce a large fraction of output as final goods, defined by being above the median industry in the economy in fraction of output that are final goods. As I am concerned with longer run trends, I want take out short run fluctuations in revenue productivity, R&D spending, and expenditure composition. I divide the sample period into bins of 5 years, and taking average values across the 5 years for each bin[3].

Consider first the relationship between the consumption from older households and the innovation gap. I run the following regression

$$Y_{jt} = \beta_0 + \beta_1 S_{jt} + \alpha_j + \eta_t + \epsilon_{jt},$$

with j denoting industry, t denoting the 5-year period, $Y_{jt} \in \{\bar{x}_{2jt} - \bar{x}_{1jt}, \bar{x}_{3jt} - \bar{x}_{1jt}, \bar{x}_{4jt} - \bar{x}_{1jt}\}$ as the innovation gap, and S_{jt} as the share of expenditures by older households. I include industry and time period fixed effects. Results are given in Table 1.

[2]Bornstein (2021) finds that households above 35 significantly less likely to switch products than those younger. Results are similar using higher age cutoffs.

[3]Results are similar when using 3 year bins instead

The consumption share of older households comoves more positively with the innovation gap for higher productivity groups, when using either R&D or $\log(\text{R\&D}+1)$ as proxy for innovation. For the gap between the $50^{th} - 75^{th}$ quantile firms and the bottom 50^{th} quantile firms, the point estimates are negative. The sign of the estimates switch to positive for the above 75^{th} quantile group and the above 90^{th} quantile group. The estimates are also larger and more statistically significant for the above 90^{th} quantile group. While not shown here, the patterns are similar when using R&D per worker in place of R&D. The estimates suggest that as the consumption share of older households increase due to demographic shift, the most productive firms are further distancing themselves from the rest[4].

Now consider the relationship between the consumption from older households and productivity dispersion. Motivated by equation (1), I run the following regression

$$\Delta Disp_{jt} = \gamma_0 + \gamma_1 S_{jt-1} + \psi_j + \zeta_t + \varepsilon_{jt},$$

where $\Delta Disp_{jt}$ is the change in revenue productivity dispersion between 5-year period $t-1$ and 5-year period t. Results are in the last column of Table 1.

When the share of consumption by older households rises in an industry, that industry sees a more positive change in their log revenue productivity dispersion over a 5 year period. For a rough calculation on the economic size of this relationship, between 1990 and 2019The share of consumption by older households increased by 0.07. Using the point estimate, with the caveat that the consumption share is not entirely exogenous, over a period of 30 years, a steady 0.07 increase in the share of consumption by older households would imply a cumulative increase of around 0.15 in log revenue productivity dispersion. This is more than what is observed in Figure 1. This suggests that demographic shift could have a sizable role in explaining the increase in revenue productivity dispersion.

3 A model of consumption habits and firm innovation

In this section, I lay out a model with consumer capital that arise through consumption habits, and explore the mechanism through which it affects firm innovation incentives and productivity dispersion as the share of older households in the economy increase. The focus on consumption habits is motivated by the results in the previous section, which suggest an effect of demographic shift that works through changes in the age composition of firm demand. Moreover, consumption habits is important in determining demand, and varies between young and older consumers (Bornstein (2021), Bronnenberg et al. (2012)).

[4]The observation of frontier firms growing faster is explored in Andrews et al. (2019)

Dep. var	R&D			log (R&D + 1)			$\Delta Disp_{jt}$
	$\bar{x}_{2jt} - \bar{x}_{1jt}$	$\bar{x}_{3jt} - \bar{x}_{1jt}$	$\bar{x}_{4jt} - \bar{x}_{1jt}$	$\bar{x}_{2jt} - \bar{x}_{1jt}$	$\bar{x}_{3jt} - \bar{x}_{1jt}$	$\bar{x}_{4jt} - \bar{x}_{1jt}$	
S_{jt}	-8.38	15.86**	33.52**	-1.03	2.59	7.82**	
	(11.71)	(7.59)	(13.72)	(2.03)	(2.52)	(3.76)	
S_{jt-1}							0.92**
							(0.41)
N	139	151	142	139	141	142	144

Table 1: Consumption from older households, the Innovation gap, and Revenue productivity dispersion

The model builds on the step-by-step innovation framework (Aghion et al. (2001)), where firms in each industry race in R&D to be the market leader. Additionally, firms accumulate consumer habits for their products (Ravn et al. (2006)). These habits boost the firm's demand, and is built from past household consumption. Households stochastically age and stochastically die, and the rate of aging and rate of death determine the share of older households in the economy. For the baseline results, I model the demographic shift as decreasing the probability of death, which effectively raises the share of older households in the economy.

3.1 Environment

3.1.1 Households

There are two types of households in the economy, young and old, with a total mass of 1. Each period, a random portion ϵ_1 of young households turn old, a random portion ϵ_2 of old households leave the economy, and new young households enter the economy. The mass of entering household is the same as the mass of leaving households. Denote the mass of young and old households by M_y and M_o respectively.
Households discount the future at rate β. Their period preferences are given by

$$U_t^a = \ln C_t^a - L_t^a,$$

where $a \in \{Y, O\}$ denotes household type, C_t^a is a consumption aggregator over sectoral goods, $C_t^a = \exp\left[\int \ln C_{jt}^a dj\right]$, and L_t^a is the household's labour supply. Their budget constraint is

$$P_t^a C_t^a + P_t^A A_{t+1}^a = L_t^a + (1 + d_t) P_t^A \left(1 + \hat{A}_t^O \frac{\epsilon_2 M_o}{\hat{A}_t^Y M_y + \hat{A}_t^O (1 - \epsilon_2) M_o}\right) A_t^a,$$

where A_t^a is a claim to a bundle of all firms in the economy, P_t^a is the price index on aggregated consumption, P_t^A is the price of the firm bundle, and d_t is the bundle's dividend payout. I normalize the wage to 1. The price index on aggregated consumption is allowed to differ by household types: while households face the same price for individual goods, differences in their habits will lead to different price indexes. I also assume that firm shares held by old households who leave the economy are redistributed to current households relative to how much shares their type hold in the aggregate, $\hat{A}_t^a$. These shares are exogenous to any individual household.

In each sector, there is a pair of duopolist along with a fringe producing imperfectly substitutable goods. The sectoral good is given by the aggregator

$$C_{jt}^Y = \left(0.5^{\frac{-\theta}{\rho}} \left[0.5^{\frac{\theta}{\rho}} \left(C_{1jt}^Y\right)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \left(C_{2jt}^Y\right)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \int^{\mathcal{N}} C_{fjt}^Y(x)^{\frac{\rho-1}{\rho}} dx\right]\right)^{\frac{\rho}{\rho-1}}$$

$$C_{jt}^O = \left(0.5^{\frac{-\theta}{\rho}} \left[k_{1jt}^{\frac{\theta}{\rho}} \left(C_{1jt}^O\right)^{\frac{\rho-1}{\rho}} + k_{2jt}^{\frac{\theta}{\rho}} \left(C_{2jt}^O\right)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \int^{\mathcal{N}} C_{fjt}^O(x)^{\frac{\rho-1}{\rho}} dx\right]\right)^{\frac{\rho}{\rho-1}},$$

where $\mathcal{N}$ is the (exogenous) mass of fringe firms, and ρ determines how substitutable goods within the sector are. Households' utility for consuming each firm's good is affected by their habit stock, with the strength of this effect governed by θ. Young households have equal habit stocks, constant at 0.5, across all goods. As such, consumption habits do not affect young households. Old households' habit stocks for goods produced by duopolists, k_{1jt} and k_{2jt}, are determined based on past expenditure on these goods. I assume that these habits are external: they are determined by the average expenditure of other old households, so that a household's own consumption does not affect their habit stock. Habit stocks evolve according to

$$k_{ijt} = (1-\delta)\,\bar{k}_{ijt-1} + \delta \frac{p_{ijt-1}\bar{c}_{ijt-1}}{p_{ijt-1}\bar{c}_{ijt-1} + p_{-ijt-1}\bar{c}_{-ijt-1}} \tag{2}$$

$$\begin{aligned}
\bar{k}_{ijt-1} &= \frac{0.5\epsilon_1 M_y + k_{ijt-1} M_o (1-\epsilon_2)}{\epsilon_1 M_y + M_o (1-\epsilon_2)} \\
\bar{c}_{ijt-1} &= \frac{C_{ijt-1}^y \epsilon_1 M_y + C_{ijt-1}^o M_o (1-\epsilon_2)}{\epsilon_1 M_y + M_o (1-\epsilon_2)}.
\end{aligned}$$

$\bar{k}_{ijt-1}$ represents the average habit stock for old households in period $t-1$, taken as the weighted average of habit stocks from young households that newly turned old, and habit stocks of surviving old households. Similarly $\bar{c}_{ijt-1}$ represents the average consumption for old households in period $t-1$. δ determines the speed at which consumer capital changes.

Note that in this setup, given starting stocks k_{ij0}, k_{-ij0} such that $k_{ij0} + k_{-ij0} = 1$, we have if $k_{ijt} + k_{-ijt} = 1 \ \forall t$.

With the given preference structure, demand for duopolist good ij at time t for an individual household, conditional on their habit stocks $\{k_{1jt}, k_{2jt}\}$, is

$$C^Y_{ijt} = \frac{p_{ijt}^{-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}$$

$$C^O_{ijt} = \frac{(2k_{ijt})^{\theta} p_{ijt}^{-\rho}}{(2k_{ijt})^{\theta} p_{-ijt}^{1-\rho} + (2k_{-ijt})^{\theta} p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}.$$

3.1.2 Firms

In each sector, firms engage in Cournot competition. Production technology is given by

$$Y_{ijt} = q_{ijt} l_{ijt},$$

where $Y_{ijt}, q_{ijt}, l_{ijt}$ is the firm's goods output, productivity, and labour input. Firm demand is given by summing up demand across households:

$$C_{ijt} = \left(C^Y_{ijt} M_y + C^O_{ijt} M_o\right).$$

Firms can invest in R&D to improve productivity probabilistically. To achieve a R&D success probability of ι_{ijt}, the firm has to employ $\frac{\gamma}{2}\left(\log\left(\frac{1}{1-\iota_{ijt}}\right)\right)^2$ units of labour in R&D. If successful, the firm's productivity increases proportionally by a factor $\lambda > 1$, so that

$$q_{ijt+1} = \lambda q_{ijt}.$$

Assuming that $q_{ij0} = 1$, we have $q_{ijt} = \lambda^{n_{ijt}}$ where n_{ijt} is the number of successful productivity improvements since time 0. The relative productivity between 2 firms within a sector is then

$$\frac{q_{ijt}}{q_{-ijt}} = \lambda^{n_{ijt} - n_{-ijt}} =: \lambda^{m_{ijt}}.$$

Here m_{ijt} denotes the technology gap between firm i and its competitor $-i$ in sector j. Assume that there is a maximal gap $\bar{m}$ such that $\bar{m} \geq m_{ijt} \geq -\bar{m}$. I also assume that the firm with lower technology has a chance of achieving a breakthrough with each successful innovation, effectively catching up to the its rival firm's technology level from the previous period. That is, when a follower i successfully innovates, with probability ϕ they close the technology gap and have $n_{ijt} = n_{-ij(t-1)}$.

Fringe firms do not innovate. I assume that their technology level is the average of duopolists' in their sector: $q_{fjt} = \lambda^{\frac{n_{ijt} + n_{-ijt}}{2}}$.

3.2 Characterization

3.2.1 Households

With the possibility of multiple types of households owning firms, it is not obvious at which rate the firms discount at. I show that in equilibrium, old households are on their Euler equation for assets, and that firms discount at rate β. So when I conduct my baseline exercise of changing ϵ_2, firms' discount rate will remain unaffected. This isolates the effect of aging demographics that operates through consumer habits, as opposed to effects that operate through the interest rate.

Proposition 1. *In equilibrium, old households' Euler equation for assets holds with equality. Firms discount at rate* β*.*

Proof. Appendix. □

3.2.2 Firms

Fringe firms cannot affect their elasticity of demand. They charge a fix markup over their marginal cost, setting price as $p_{fjt}(x) = \frac{1}{q_{fjt}} \frac{\rho}{\rho - 1}$.

Duopolists can affect their demand and demand elasticity, through the quantity that they currently produce and through their consumer capital. Duopolist profit is given by

$$\pi_{ijt} = p_{ijt} C_{ijt} - l_{ijt} = s_{ijt} - l_{ijt},$$

where I have defined $s_{ijt} \equiv p_{ijt} C_{ijt}$. Since sectoral expenditure $p_{jt} C_{jt} = 1$, s_{ijt} is also the expenditure share of good ij in sector j at time t. As firms compete in quantities, their choice variable is effectively L_{ijt}. (s_{ijt}, s_{-ijt}) can be solved as an implicit function of l_{ijt} and l_{-ijt}, along with the productivity gap m_{ijt} between the duopolists.

The problem of a duopolist can be written recursively as

$$\begin{aligned} v(k, k_-, m) = & \max_{l,\iota} s(l, l_-, k, k_-, m) - l - \frac{\gamma}{2} \left(\log \left(\frac{1}{1-\iota} \right) \right)^2 \\ & + \beta E_{m'} \left[v(k', k'_-, m') \right] \end{aligned} \tag{3}$$

with k' evolving according to equation (2).

3.3 Balanced growth path equilibrium

I consider the recursive equilibrium on the balanced growth path (BGP) of the economy, where household mass M_y, M_o are constant, aggregate consumption C^Y, C^O grow at a constant rate, and the distribution of sectors is stationary. Formally, the recursive equilibrium on the BGP consists of household policies $\left\{C^Y(k,k_-,m), C^O(k,k_-,m), A^Y, A^O, L^Y, L^O\right\}$, firm policies $\{l(k,k_-,m), \iota(k,k_-,m)\}$, firm value $v(k,k_-,m)$, distribution of sectors $\Omega(k,k_-,m)$, law of motion Γ for $\Omega(k,k_-,m)$, and (relative) prices $\left\{P^A, \frac{p_-}{p}(k,k_-,m), \frac{p_f}{p}(k,k_-,m)\right\}$, such that

1. $\left\{C^Y(k,k_-,m), C^O(k,k_-,m), A^Y, A^O, L^Y, L^O\right\}$ solves the household problem, given prices
2. Given competitor's policies $\{l(k,k_-,m), \iota(k,k_-,m)\}$, the firm value $v(k,k_-,m)$ is consistent with the firm Bellman equation (3), and firm policies $\{l(k,k_-,m), \iota(k,k_-,m)\}$ are consistent with maximization
3. P^A clears the asset market
4. Relative prices $\left\{\frac{p_-}{p}(k,k_-,m), \frac{p_f}{p}(k,k_-,m)\right\}$ clears the goods market for each sector
5. The distribution of sectors $\Omega(k,k_-,m)$ is stationary, and its law of motion, Γ, is consistent with firm policies: For all sets S in the Borel algebra of the domain of Ω, and for all states (k,k_-,m) with $k+k_-=1$,

$$\Omega(S) = \int \left\{1_{\{(k'(k,k_-,m),1-k'(k,k_-,m),m\prime)\in S\}} Pr\left(m'|\iota(k,k_-,m), \iota(k_-,k,-m), m\right)\right\} d\Omega(k,k_-,m)$$

The equilibrium concept is standard, with the addition that duopolist behavior within a sector is strategic and constitutes a Markov Perfect Equilibrium.

3.4 Model mechanisms

To illustrate the mechanism in in which aging demographic affects productivity dispersion through consumer capital, I analyze a stripped down version of the quantitative model. Consider a sector for only 2 periods, where in the final period firms only care about profit. Assume that there is no fringe ($\mathcal{N}=0$), that the probability of a breakthrough innovation by the follower is 0 ($\phi=0$), and that the cost of R&D is $\frac{\gamma}{2}\iota^2$ (a good approximation to $\frac{\gamma}{2}\left(\log\left(\frac{1}{1-\iota}\right)\right)^2$ for ι near 0). Furthermore, assume a representative consumer that build up external habits through past expenditures.

In the quantitative model, increasing the mass of old households in the economy effectively increases the importance of consumer capital in the firm's demand. With this stripped down version, I model the effect of aging demographics as increasing the strength of external habits θ.

Firms now face inverse demand

$$p_{it} = \frac{c_{it}^{-1/\rho}}{c_{it}^{\frac{\rho-1}{\rho}} + \left(\frac{k_{-i}}{k_i}\right)^{\theta/\rho} c_{-it}^{\frac{\rho-1}{\rho}}},$$

with $k_{i2} = p_{i1}y_{i1}$ and k_{i1} given. This implies an elasticity of demand

$$\varepsilon_{it} = \left[\frac{1}{\rho} + \frac{\rho-1}{\rho}\frac{c_{it}^{\frac{\rho-1}{\rho}}}{c_{it}^{\frac{\rho-1}{\rho}} + \left(\frac{k_{-i}}{k_i}\right)^{\theta/\rho} c_{-it}^{\frac{\rho-1}{\rho}}}\right]^{-1}.$$

Let firm i be the firm that is more productive. This firm is also most likely to be the firm with higher consumer capital ($k_{it} > 0.5$). Fixing the level of consumer capital, an increase in θ both increases demand for the firm and make demand more inelastic.

First, the expected change in log productivity dispersion from period 1 to period 2 is proportional to the expected change in the technology gap m. This change can be approximated by

$$E\left[m_2\right] - m_1 = \iota_{i1} - \iota_{-i1}.$$

Expected productivity dispersion increases if the more productive firm innovates more than the less productive firm, and the size of the increase is proportional to the R&D investment gap. It is through affecting the R&D investment gap, that aging demographics affects productivity dispersion.

Now consider the individual innovation choices. They can be approximated by

$$\begin{aligned}
\gamma\iota_{i1} &= \beta\left[\pi_2\left(k_{i2}, m+1\right) - \pi_2\left(k_{i2}, m\right)\right] \\
\gamma\iota_{-i1} &= \beta\left[\pi_2\left(1-k_{i2}, -m+1\right) - \pi_2\left(1-k_{i2}, -m\right)\right].
\end{aligned}$$

The left hand side gives marginal the cost of R&D. The right hand side is the marginal benefit, which is proportional to the gain from increasing productivity by a single step. Final period profits is given by

$$\pi_2\left(k, m\right) = \frac{\left(\frac{k}{1-k}\right)^{\theta/\rho}\lambda^{m(\rho-1)/\rho}\left(\left(\frac{k}{1-k}\right)^{\theta/\rho}\lambda^{m(\rho-1)/\rho} + \frac{1}{\rho}\right)}{\left(1 + \left(\frac{k}{1-k}\right)^{\theta/\rho}\lambda^{m(\rho-1)/\rho}\right)^2}. \tag{4}$$

Movements in θ will affect the gain from increasing productivity by a single step. The following proposition gives a more precise characterization of the effect.

Proposition 2. *For* π_2 *as in equation 4, let* $x = \left(\frac{k}{1-k}\right)^{\theta/\rho}$*. Then,*

a) For $x < 1$ *and* $m < 0$, $\frac{\partial^2 \pi}{\partial m \partial x} > 0$.

b) For $x > 1$ *and* $m > 0$*, the sign of* $\frac{\partial^2 \pi}{\partial m \partial x}$ *is ambiguous. It is positive for* $x\lambda^{m(\rho-1)/\rho}$ *close to 1.*

Proof. Appendix. □

Part (a) of proposition 2 concerns the follower, with k_{-i2} likely to be less than 0.5, so $x_{-i} = \left(\frac{k_{-i2}}{1-k_{-i2}}\right)^{\theta/\rho} < 1$. An increase in θ decreases x_{-i}, which decreases the gain from increasing productivity $\frac{\partial \pi}{\partial m}$. So then an increase in θ decreases the follower's innovation incentives. Part (b) concerns the leader, with x_i likely to be more than 1. An increase in θ increases x_i, and would increase the leader's innovation incentives if $\frac{\partial^2 \pi}{\partial m \partial x}$ is positive. If this was the case, the investment gap $\iota_{i1} - \iota_{-i1}$ would unambiguously increase, resulting in increased productivity dispersion.

Decreased productivity dispersion is also possible when $\frac{\partial^2 \pi}{\partial m \partial x}$ is negative. The ambiguity in how the habit parameter θ affects productivity dispersion is because an increase in θ both increases demand for the leader and make its demand more inelastic. The increase in demand implies the firm can charge more additional unit produced, which boosts the gain from higher productivity as the firm can produce more for lower cost. But lower elasticity implies that the firm could increase prices by restricting output, hence reducing the gain from increased production.

Which effect dominates is determined by the term $y_i \equiv \left(\frac{k_{i2}}{1-k_{i2}}\right)^{\theta/\rho} \lambda^{m(\rho-1)/\rho}$, which is closely related to the degree of market power the leader has. y_i close to 1 implies that the two firms are close to being neck-in-neck in competition, with market shares at around 0.5 for each firm. For $\rho = 10$, the value used for calibration in the next section, the upper bound on y_i for $\frac{\partial^2 \pi}{\partial m \partial x}$ to be positive is 1.946. This is not too restrictive: for example, with $\theta = 1.5$ and $k_{i2} = 0.8$, the log productivity difference between the leader and follower would have to be higher than 0.51 to hit the bound.

3.5 Parameterization

The quantitative model has 10 parameters $(\beta, \epsilon_1, \epsilon_2, \lambda, \gamma, \phi, \mathcal{N}, \rho, \theta, \delta)$. Table 2 summarizes the parameter choices for the model. I set the discount rate β at 0.96. ϵ_1^{-1} and ϵ_2^{-1} maps to the mean age of young and old households respectively. I set ϵ_1 at 15^{-1}, so that, assuming households enter the economy at 20 years old, young households are on average 35 years old. I set ϵ_2 so that ϵ_2^{-1} corresponds to the mean age of households 35 and over for the US, minus 35, in the 1960s. This is around 18 years.

I set λ, the innovation step size, to target a TFP growth rate of 1.7%. γ, which governs the cost of R&D, is set to target a total R&D spending to GDP of 4% . I set ϕ, the probability of a breakthrough innovation from the less productive firm, to achieve 0.2 revenue productivity dispersion.

For θ and δ, the parameters governing the strength and depreciation of consumer habit, I rely on estimates from Bronnenberg et al. (2012). The authors leverage migration as a source of change in the market conditions facing the individuals migrating, and uses changes in consumption patterns to inform the strength and depreciation of consumer habit. Consider two markets in different locations, A and B, with goods x and y as the two products with the highest market share in both markets. Define the relative share of good x as the ratio of x's market share to the sum of x and y's market share. Assume that relative prices of x and y differ between the two markets[5], leading to different relative market shares s_x^A and s_x^B. Suppose an individual i migrates from A to B. The extent to which s_x^i, i's relative expenditure shares on x, looks similar to s_x^A as opposed to s_x^B upon migration will inform the strength of consumption habits. Define $G \equiv \frac{s_x^i - s_x^A}{s_B^i - s_x^A}$ to capture this extent. If there was no consumption habit, we would expect s_x^i to be the same as long-time residents in B, so $G = 1$. Whereas if consumption habits were perfectly rigid, we would have $G = 0$. Moreover, the time it takes after migration for G to rise to 1 will inform the depreciation of consumption habits.

Bronnenberg et al. (2012) finds that 60% of the difference in long term expenditure shares is reached when an individual migrates. The remaining 40% is closed in subsequent years, with a half-life of 22 years. This is equivalent to the following setting. Consider 2 firms in a market with equal long term prices, hence equal relative market shares at 0.5. Suppose one of the firms decreases its price so that its new long term relative market share would be 0.6. Upon the price change, that firm would see its relative market share rise to 0.56, or 60% of the gap.

For δ, I choose a more conservative value of 0.05, which correspond to a half-life of 10 years. I then replicate the price change experiment: starting in a sector with $k = k_- = 0.5$ and $m = 0$, I change m to 1 and move prices according to equilibrium firm policies. θ is set so that the instantaneous change in expenditure shares covers 65% of the difference in long run expenditure shares. I purposefully set the targets for δ and θ to be more conservative that the estimates in Bronnenberg et al. (2012). The baseline results are stronger with a lower δ

[5]This could be from differences in supply costs or regional contracts.

and higher θ.

Param.	Description	Value	Param.	Description	Value
β	Discount rate	0.96	$\mathcal{N}$	Mass of fringe	3
ϵ_1	Prob. of becoming old	15^{-1}	θ	Strength of consumer habit	1.5
ϵ_2	Prob. of death	18^{-1}	δ	Depreciation of consumer habit	0.05
λ	Growth step size	1.135	γ	Cost of R&D	2.5
ρ	Sectoral elas. of substitution	10	ϕ	Prob. of breakthrough	0.5

Table 2: Parameter choices

Table 3 shows model moments under the parameter choices, compared to their targets. The parameterization right now is not so precise, only around the ballpark of the targets. A more careful calibration will be conducted in the future.

Moment	Target	Model
Revenue productivity dispersion	0.2	0.2
Fraction of long term market share obtained upon price change	0.65	0.69
Mean markups	1.25	1.26
TFP growth rate	1.7%	2.0%
R&D intensity	4%	3.6%
Mean market share	0.3	0.3

Table 3: Model moments

3.6 Baseline results

To model the effect of demographic shift, I compare the model economy with the above parameterization to one where $\epsilon_2 = 22^{-1}$. This value of ϵ_2 corresponds to the mean age of households 35 and over for the US, minus 35, in the 2010s, which is around 22 years. The results are shown in Table 4.

In the model, the demographic shift increases revenue productivity dispersion from 0.2 to 0.246. This is around half of the observed increase in revenue productivity dispersion in the data, from the 1980s to 2010s. Accompanying this is an increase in underlying productivity dispersion, from 0.5 to 0.57. This implies that there are more sectors where the most productive firm have a large technology gap over its rival, which allows it to hold large market

Share of older households	Baseline	Higher
revenue productivity dispersion	0.2	0.246
TFP dispersion	0.498	0.567
Average markups	1.257	1.29
Average market share	0.3	0.318
R&D intensity	3.62%	3.75%
TFP growth rate	1.99%	2.02%
Difference in R&D spending	0.0256	0.0283

Table 4: Baseline results

shares and charge higher markups. As a result, average markups rises in the model, from 1.26 to 1.29, though the magnitude is less than that reported by De Loecker et al. (2020) Average market share for duopolist is also higher, from 0.3 to 0.32, similar in magnitude to that reported by Autor et al. (2020). Firms increase their R&D spending on average, with the majority of increase coming from the more productive firms. The TFP growth rate becomes slightly higher.

An analysis of the economy in the transition to a higher share of older households would be more suitable, since it is hard to argue that the economy is on a new BGP in the 2010s. I am working to incorporate this into the paper.

3.7 Measured firm risk

The increase in revenue productivity dispersion from aging demographics is a result of endogenous changes in firms' decisions in response to an exogenous change. A related question is whether exogenous factors have changed that directly increase revenue productivity dispersion. One such factor is higher firm risk, or increased variance in productivity shocks to firms. One way the literature have tried to measure firm risk is by backing out innovations to firm productivity from an AR regression, and evaluating the variance of those innovations.

Specifically, consider the regression

$$a_{it} = \rho a_{it-1} + \gamma_i + \xi_t + \varepsilon_{it}, \tag{5}$$

where a_{it} is measured log revenue productivity for firm i at date t. We are interested in the dispersion of the predicted residuals $\hat{\varepsilon}_{it}$ from this regression over time. Following Bloom et al. (2018), I use the interquartile range as the measure of dispersion. Figure 3 plots the dispersion over time, along with a fitted time trend.

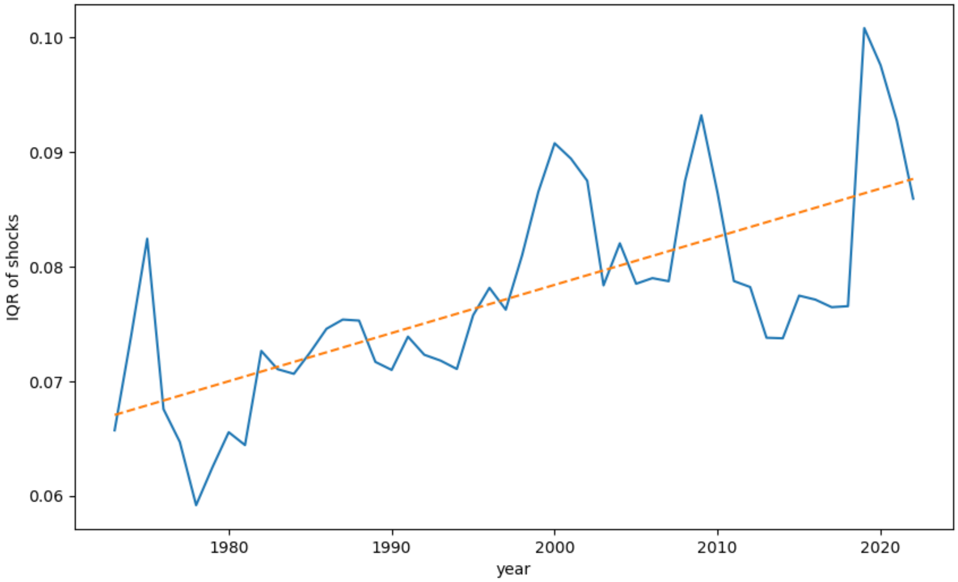

Figure 3: Interquartile range of innovations to revenue productivity

The literature has documented the counter-cyclical properties of the dispersion of shocks. But there is a rising time trend as well, with dispersion going up by 0.014 from 1980 to 2019. This might indicate that shocks to firms have increased in variance, contributing to increased revenue productivity dispersion. It also increases firm risks, potentially depressing investment and output.

However, the increased dispersion of measured shocks may just stem from increases in firm innovation rates. In the model, the standard deviation of a duopolist's log TFP tomorrow, conditional on TFP today, can be approximated by

$$(a_{ijt+1} - \mathbb{E}a_{ijt+1}|a_{ijt})^2 = \lambda^2 \left(\iota_{ijt} + \iota_{-ijt}\left(1-\phi\right) + \iota_{-ijt}\phi m\right), \tag{6}$$

which is increasing in innovation rates and productivity dispersion. The model implied an increase in R&D spending in response to aging demographics as well as higher productivity dispersion, so the conditional variance of productivity would increase as well. Measured shocks would then increase, under the condition that the persistence of productivity ρ does not change much over time.

First, I check the increase in variance of measured shocks in the model. I simulate a panel of firms and run the regression 5 on the simulated panel. There, demographic shift generates an increase in the interquartile range of shocks from 0.091 to 0.106. This is similar to the observed increase in the data of 0.072 to 0.087.

Second, equation (6) motivates a regression of the variance of measured shocks with controls on endogenous objects, to see whether the variance has increased over time. I run the following regression

$$\sigma_t = \beta X_t + \gamma t + \nu_t,$$

where σ_t is the interquatile range of recovered shocks in time t, and X_t is a set of controls of endogenous objects. X_t includes lagged aggregate revenue productivity dispersion, lagged average R&D spending of all firms, lagged average R&D spending of firms in the $50^{th} - 75^{th}$ quantile of productivity, lagged average R&D spending of firms in the $75^{th} - 100^{th}$ quantile of productivity, and lagged average standard deviation of R&D spending in each industry. Results are given in table 5.

	No controls	Full controls
t	4e-4***	-6e-4
	(6e-5)	(4e-4)
Adj. R2	0.460	0.566
N	50	50

Table 5: Regression on interquartile range of recovered shocks

The results suggest that the exogenous volatility of shocks have not gone up. With no controls other than a constant, the coefficient on the time trend γ is positive and significant. After including the set of controls, the coefficient becomes negative and no longer significant. The adjusted R-squared also goes up, indicating that the set of controls contribute to the explanatory power.

4 Conclusion

This paper explored the link between aging demographics and rising revenue productivity dispersion in the US. Data on expenditure shares of older households and revenue productivity dispersion within industries suggests that as the population becomes older, there could be a sizable increase in revenue productivity dispersion. I then build a model to understand the mechanism that drives this, and to further quantify the effect.

The model works through consumer capital. A rise in the share of older consumers both increases demand and makes demand more inelastic for firms that are more productive and have high consumer capital, while decreasing demand for firms that are less productive and have low consumer capital. High productivity firms would then either increase R&D investment to increase production and meet higher demand, or decrease R&D investment in favor of charging higher markups. Meanwhile low productivity firms would decrease R&D investment due to decreased demand.

Setting parameters so that the baseline model economy matches the US in the 1970s, I then increase the longevity of old households to model aging demographics. I compare the resulting economy to the baseline. For the given parameters, the gap in R&D spending between more productive firms and less productive ones widen, so that productivity dispersion increases. revenue productivity rises from 0.2 to 0.246 in the model, around half of what observed in the data. Along with this is an increase in average markups and average market share of top firms, which is consistent with the data.

An important feature that could be incorporated into the model is firm entry. Entrants most often start as followers in an industry, so the mechanism that generates increased productivity dispersion should lower the value of being a follower, hence lower the entry rate. The inclusion of entry would allow the model to speak to the decline in the entry rate observed since the 1980s.

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A revenue productivity estimation

I estimate revenue productivity following Flynn et al. (2019). The approach uses a proxy estimator to estimate the production function (Ackerberg et al. (2015)), but with an additional restriction on returns to scale, which is necessary for identification. I assume a translog production function

$$y_{it} = \theta_t^v v_{it} + \theta_t^k k_{it} + \theta_t^{vv} v_{it}^2 + \theta_t^{kk} k_{it}^2 + \theta_t^{vk} v_{it} k_{it} + a_{it} + \epsilon_{it},$$

where y_{it} is log revenue, v_{it} is log cost of goods sold, k_{it} is log capital, and a_{it} is log revenue productivity. As in De Loecker et al. (2020), I allow for time-varying production function parameters, and estimate separately for each 2 digit NAICS sector.

k_{it} and v_{it} may be correlated with a_{it}, which gives rise to a simultaneity problem if we proceed to estimate the above function via OLS. The key insight is that a_{it} can be expressed as a function of the firm's observables, obtained from inverting out input demand:

$$a_{it} = \omega_t \left(v_{it}, k_{it}, z_{it} \right),$$

where z_{it} captures other factors that affect demand. Output can then be written as

$$y_{it} = \phi_{it} \left(v_{it}, k_{it}, z_{it} \right) + \epsilon_{it}.$$

For a given guess of $\theta_t = \left\{ \theta_t^v, \theta_t^k, \theta_t^{vv}, \theta_t^{kk}, \theta_t^{vk} \right\}$, one can obtain a guess of revenue productivity as

$$\tilde{a}_{it} \left(\theta_t \right) = \phi_{it} \left(v_{it}, k_{it}, z_{it} \right) - \left(\theta_t^v v_{it} + \theta_t^k k_{it} + \theta_t^{vv} v_{it}^2 + \theta_t^{kk} k_{it}^2 + \theta_t^{vk} v_{it} k_{it} \right).$$

I assume a Markov productivity process $a_{it} = g \left(a_{it-1}, \hat{\psi}_{it-1} \right) + \eta_{it}$, where $\hat{\psi}_{it-1}$ is the predicted probability that the firm continues to be in the sample. This gives one moment condition for θ_t:

$$\mathbb{E} \left[k_{it} \eta_{it} \right] = 0.$$

I impose the additional conditions that the return to scale is 1, which gives 3 more moments:

$$\begin{aligned} \mathbb{E} \left[v_{it} \left(RTS_{it} \left(\theta_t \right) - 1 \right) \right] &= 0 \\ \mathbb{E} \left[k_{it} \left(RTS_{it} \left(\theta_t \right) - 1 \right) \right] &= 0 \\ \mathbb{E} \left[\left(RTS_{it} \left(\theta_t \right) - 1 \right) \right] &= 0, \end{aligned}$$

where $RTS_{it} \left(\theta_t \right) = \theta_t^v + \theta_t^k + 2\theta_t^{vv} v_{it} + 2\theta_t^{kk} k_{it} + \theta_t^{vk} v_{it} k_{it}$.

B Proofs and derivations

Proposition. *In equilibrium, old households Euler equation for assets holds with equality. Firms discount at rate β.*

Proof. Households Euler equations are given by

$$P_t^Y C_t^Y = 1$$
$$P_t^O C_t^O = 1$$

$$\beta \frac{P_{t+1}^A \left(1 + d_{t+1}\right) \left(1 + \hat{A}_{t+1}^O \frac{\epsilon_2 M_o}{\hat{A}_{t+1}^Y M_y + \hat{A}_{t+1}^O (1-\epsilon_2) M_o}\right)}{P_t^A} \left(\epsilon_1 \frac{1}{C_{t+1}^O P_{t+1}^O} + \left(1 - \epsilon_1\right) \frac{1}{C_{t+1}^Y P_{t+1}^Y}\right) \geq \frac{1}{C_t^Y P_t^Y}$$
$$\beta \frac{P_{t+1}^A \left(1 + d_{t+1}\right) \left(1 + \hat{A}_{t+1}^O \frac{\epsilon_2 M_o}{\hat{A}_{t+1}^Y M_y + \hat{A}_{t+1}^O (1-\epsilon_2) M_o}\right)}{P_t^A} \left(\left(1 - \epsilon_2\right) \frac{1}{C_{t+1}^O P_{t+1}^O}\right) \geq \frac{1}{C_{t+1}^O P_{t+1}^O}$$

Rearranging the Euler equation for assets:

$$\beta \frac{P_{t+1}^A \left(1 + d_{t+1}\right) \left(1 + \hat{A}_{t+1}^O \frac{\epsilon_2 M_o}{\hat{A}_{t+1}^Y M_y + \hat{A}_{t+1}^O (1-\epsilon_2) M_o}\right)}{P_t^A} \geq 1$$
$$\beta \frac{P_{t+1}^A \left(1 + d_{t+1}\right) \left(1 + \hat{A}_{t+1}^O \frac{\epsilon_2 M_o}{\hat{A}_{t+1}^Y M_y + \hat{A}_{t+1}^O (1-\epsilon_2) M_o}\right)}{P_t^A} \left(1 - \epsilon_2\right) \geq 1$$

It must then be that old households are on their Euler equation for assets, and young households do not hold asset. So $\hat{A}_{t+1}^Y = 0, \hat{A}_{t+1}^O = 1$ in equilibrium, so that

$$\beta \frac{P_{t+1}^A \left(1 + d_{t+1}\right)}{P_t^A} = 1,$$

hence firms discount at rate β. □