

Customer capital and firm innovation

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Introduction

- Customer capital: Value from customer base through repeated transactions
- Important form of intangible capital:
 - ▶ Linked to firm valuation, sales, lower default risk
 - ▶ Large expense on advertising and sales; Affects how firm set prices >
- Paper goal: Study effect of customer capital on firm decision to innovate
 - ▶ Important for understanding trends in productivity dispersion, concentration, markups
 - ▶ Framework to evaluate innovation subsidies, changing consumption patterns

This paper

- Develop model where
 - ▶ Firms innovate to reduce cost; Customer capital arise from consumption habits, where older households have stronger habits
 - ▶ Model makes predictions on how strength of customer capital affects R&D spending and productivity dispersion
 - ▶ Validate using industry age composition of demand

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 - ▶ Validate using industry age composition of demand
- Motivated by higher consumption persistence for older households
 - ▶ Quantify effect of aging demographics: Generates 10%-35% of observed movements in R&D spending differences, concentration, markups

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 - ▶ Validate using industry age composition of demand
- Motivated by higher consumption persistence for older households
 - ▶ Quantify effect of aging demographics: Generates 10%-35% of observed movements in R&D spending differences, concentration, markups
- Innovation subsidies with customer capital
 - ▶ Amplified effect on concentration and markups, 2-3 times greater than without customer capital

Literature

- Customer capital:
 - ▶ Larkin (2013), Gourio and Rudanko (2014), **Foster et. al. (2016)**, Baker et. al. (2023), Afrouzi et. al. (2023) \Rightarrow Effect on firm innovation
- Intangibles and innovation:
 - ▶ Cavenaile and Roldan-Blanco (2020), **Cavenaile et. al. (2023)**, Shen (2023), De Ridder (2024) \Rightarrow Persistent customer capital + competition structure
- Accounting for aggregate trends in productivity dispersion, concentration, markups:
 - ▶ Karahan et. al. (2019), Peters and Walsh (2021), **Bornstein (2021)**, **Olmstead-Rumsey (2022)**, **Akcigit and Ates (2023)** \Rightarrow Complementary demand mechanism

Outline

- Simple model
- Quantitative model
- Calibration
- Empirical support
- Effect of aging demographics
- Innovation subsidies with customer capital

Simple model

Simple model

- Two period duopoly
- First period: No production. Firm $i \in \{1, 2\}$ comes in with productivity \hat{q}_i , invest in R&D ι_i to increase productivity in second period
 - ▶ Second period productivity $q_i = \begin{cases} \lambda \hat{q}_i & \text{with probability } \iota_i \\ \hat{q}_i & \text{with probability } 1-\iota_i \end{cases}$
 - ▶ Cost of R&D: $\frac{\gamma}{2} \iota_i^2$
- Second period: Cournot competition, marginal cost $1/q_i$

Simple model

- Unit mass households, 1 unit of endowment to spend ➤

- ▶ Preference: $\left(k_1^{\frac{\theta}{\rho}} c_1^{\frac{\rho-1}{\rho}} + k_2^{\frac{\theta}{\rho}} c_2^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$

- ★ Habits/Customer capital (k_1, k_2) ; Habit strength θ

- ▶ Inverse demand:

$$p_i = \frac{(k_i)^{\theta/\rho} c_i^{-1/\rho}}{(k_i)^{\theta/\rho} c_i^{\frac{\rho-1}{\rho}} + (k_{-i})^{\theta/\rho} c_{-i}^{\frac{\rho-1}{\rho}}}$$

- ▶ More customer capital k_i raise demand & reduce demand elasticity

Firm problem

- Second period:
 - ▶ Cournot game with payoff $\pi_i = (p_i - 1/q_i) c_i$
 - ▶ \Rightarrow Equilibrium payoffs $\pi(k_i/k_{-i}, q_i/q_{-i})$ >
- First period:
 - ▶ Approximation of FOC

$$\iota_i \approx \frac{1}{\gamma} [\pi(k_i/k_{-i}, \lambda \hat{q}_i/\hat{q}_{-i}) - \pi(k_i/k_{-i}, \hat{q}_i/\hat{q}_{-i})]$$

Innovation rates and customer capital

Proposition

An increase in $(k_i/k_{-i})^\theta$ increases ι_i iff $F(\rho)$

$$\underbrace{\left(\frac{k_i}{k_{-i}}\right)^{\frac{\theta}{\rho}} \left(\frac{\dot{q}_i}{\dot{q}_{-i}}\right)^{\frac{\rho-1}{\rho}}}_{\text{Relative revenue productivity}} < \underbrace{F(\rho)}_{\text{Known function}}$$

- Customer capital have opposing effects on innovation
 - ▶ Higher demand \Rightarrow Produce more \Rightarrow Larger cost reduction from innovation
 - ▶ Lower elasticity \Rightarrow Restrict supply for markups \Rightarrow Lower innovation
- Total effect depends on relative revenue productivity

Takeaway - effect of customer capital

- Relative revenue productivity $< 1 < F(\rho)$ for follower \Rightarrow innovation moves with customer capital
- For leader:
 - ▶ With reasonable ρ , $\log F(\rho) \approx 0.66$; Std of log revenue prod. ≈ 0.28 for public firms
 - ▶ \Rightarrow innovation moves with customer capital
- With stronger habits ($\theta \uparrow$), innovation increase for leader $\left((k_i/k_{-i})^\theta \uparrow\right)$ and decrease for follower $\left((k_{-i}/k_i)^\theta \downarrow\right)$
 - ▶ \Rightarrow Leader increase productivity gap $((q_i/q_{-i}) \uparrow)$, capture more market share, charge higher markups

Takeaway - what the static model misses

- With dynamics and endogenous customer capital,
 - ▶ Leader produce more \Rightarrow accumulate more $k_i \Rightarrow$ increase innovation \Rightarrow larger $\frac{q_i}{q_{-i}} \Rightarrow$ leader produce more
 - ▶ Amplified movements in productivity gap
 - ▶ Amplified effect of innovation subsidy on concentration

Quantitative model

Quantitative model

- Dynamic duopoly, continuum of industries
- Two types of households, young and old >
- Habit evolution: Accumulated past expenditure of average old household >
- Mass of fringe firms in addition to two dominant firms >
- Follower have additional chance to catch up >
- Entrant replacing follower > calib

Households

- Unit mass. Young \rightarrow old with probability ϵ^Y . Old \rightarrow dropout with probability ϵ^O ; replaced by young household
 - ▶ Mass of young and old: M^Y, M^O
- Consume goods by duopolist + continuum of fringe of mass \mathcal{N}
- Preferences

$$U_t^a = \ln C_t^a - L_t^a$$

C_t^a : nested CES, outer nest elasticity of 1, inner nest elasticity of ρ [back](#)

Households - Demand

- Firm i , sector j , time t
- Household demand for good ijt [alternative](#) [back](#):

$$C_{ijt}^Y = \frac{p_{ijt}^{-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx}$$

$$C_{ijt}^O = \frac{(k_{ijt})^\theta p_{ijt}^{-\rho}}{(k_{ijt})^\theta p_{-ijt}^{1-\rho} + (k_{-ijt})^\theta p_{-ijt}^{1-\rho} + (0.5)^\theta \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx}$$

Households - Demand

- Firm i , sector j , time t
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- Habits k_{ijt} affect old consumption, increases demand, decreases elasticity

Households - Habits

- Habits evolution [back](#)

$$\begin{aligned} [\text{Stock of habits tomorrow}] = & (1 - \delta) [\text{Stock of habits today}] + \\ & \delta [\text{Relative expenditures today}] \end{aligned}$$

Households - Habits

- Habits evolution [back](#)

$$k_{ijt+1} = (1 - \delta) \frac{\overbrace{0.5\epsilon^Y M^Y + k_{ijt} M^O (1 - \epsilon^O)}^{\text{Stock of habits today}}}{\epsilon^Y M^Y + M^O (1 - \epsilon^O)} + \underbrace{\delta \left[\frac{p_{ijt} C_{ijt}^Y}{p_{ijt} C_{ijt}^Y + p_{-ijt} C_{-ijt}^Y} \epsilon^Y M^Y + \frac{p_{ijt} C_{ijt}^O}{p_{ijt} C_{ijt}^O + p_{-ijt} C_{-ijt}^O} M^O (1 - \epsilon^O) \right]}_{\text{Relative expenditures today}} \frac{1}{\epsilon^Y M^Y + M^O (1 - \epsilon^O)}$$

- Average of **young households turning old tomorrow** and **old households alive tomorrow**

Firms

- Duopolists compete in quantities [back](#)
 - ▶ For variable x , denote leader with \bar{x} and follower with \underline{x}
- Duopolist invest in R&D to increase productivity next period:
 - ▶ Production: $Y_{ijt} = q_{ijt}l_{ijt}$
 - ▶ Leader productivity: $\bar{q}_{jt+1} = \bar{D}_{jt}\lambda\bar{q}_{jt} + (1 - \bar{D}_{jt})\bar{q}_{jt}$; $\bar{D}_{jt} = 1$ with prob. \bar{l}_{jt}

Firms

- Duopolists compete in quantities back
 - ▶ For variable x , denote leader with \bar{x} and follower with \underline{x}
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 - ▶ Follower productivity: $\underline{q}_{jt+1} = \underline{D}_{jt} (1 - \Phi) \lambda \underline{q}_{jt} + \underbrace{\underline{D}_{jt} \Phi \bar{q}_{jt}}_{\text{Closing the gap}} + (1 - \underline{D}_{jt}) \underline{q}_{jt}$;
 $\underline{D}_{jt} = 1$ with prob. \underline{l}_{jt} ; $\Phi = 1$ with prob. ϕ
 - ▶ Fringe productivity: $q_{fjt} = \bar{q}_{jt}^\alpha \underline{q}_{jt}^{1-\alpha}$
- Cost of R&D: $\frac{\gamma}{2} \left(\log \left(\frac{1}{1 - \underline{l}_{ijt}} \right) \right)^2$

Firms - Entrants

- Prospective entrant in each sector each period
- Conducts R&D to innovate on the follower's technology
- If productivity higher than the follower's (ie successful innovation), replace the follower
 - ▶ Inherit follower customer capital stock [back](#) [bm](#)

Calibration

Model parameterization

- Model calibrated to match moments from US in 1980

| Param | Description | Value | Param | Description | Value |
|--------------|--------------------------------|--------|---------------|-----------------------------------|-------|
| External | | | Internal | | |
| β | Discount rate | 0.99 | λ | Growth step size | 1.065 |
| ϵ^Y | Prob. of turning old | 0.0357 | \mathcal{N} | Mass of fringe | 6.5 |
| ϵ^O | Prob. of death | 0.0192 | α | Fringe productivity weight | 0.808 |
| ρ | Sectoral elas. of substitution | 10 | γ | Cost of R&D | 4.05 |
| δ | Depreciation of consumer habit | 0.0133 | ϕ | Prob of closing gap, upon success | 0.212 |
| | | | θ | Strength of consumer habit | 2.2 |

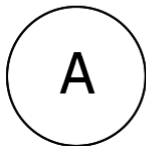
Model moments

| Moment | Model | Target | Source |
|--|-------|--------|----------------------------|
| Revenue productivity dispersion | 0.203 | 0.20 | Compustat |
| Relative change in market share after price change | 0.677 | 0.68 | Bronnenberg et. al. (2012) |
| Aggregate markups | 1.281 | 1.28 | Compustat |
| Growth rate | 2.22% | 2.2% | SF Fed |
| Mean market share | 0.265 | 0.26 | Mongey (2021) |
| Entry rate | 1.87% | 1.82% | BDS |

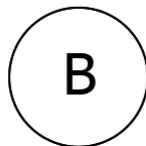
Disciplining habit parameters

- Markets A, B with goods x, y ; Market share S_A^x, S_B^x

S_A^x

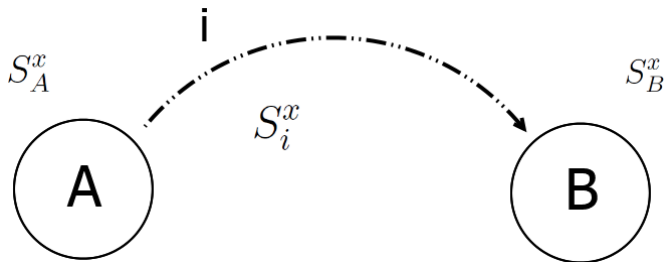


S_B^x



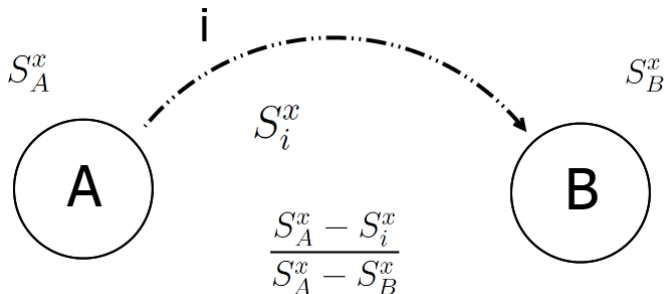
Disciplining habit parameters

- i moves $A \rightarrow B$, track i 's expenditure share S_{it}^x
- Before move, $S_{i0}^x = S_A^x$; Over time, $\lim_{t \rightarrow \infty} S_{it}^x = S_B^x$



Disciplining habit parameters

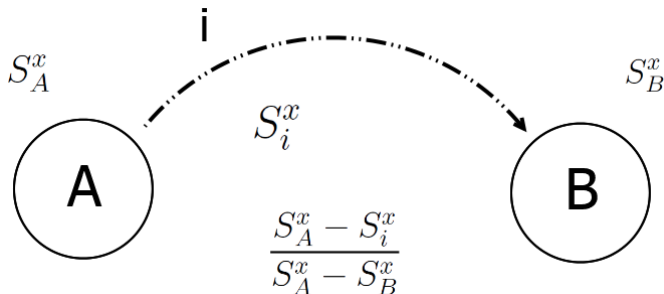
- $\frac{S_A^x - S_{i1}^x}{S_A^x - S_B^x}$ - period after move - informs strength of habits: Closer to 0 \Rightarrow stronger habits
- How fast $\frac{S_A^x - S_{it}^x}{S_A^x - S_B^x}$ converges to 1 over time informs persistence



Disciplining habit parameters - Market analog

- Start at long run A . Market conditions (ie price) changes s.t. new long run

- is B
 $\frac{S_A^x - S_{i1}^x}{S_A^x - S_B^x}$ informs strength of habits implement



Empirical support

Empirical support

- Run analysis at industry level:
 - ▶ Relationship between customer capital and innovation efforts across leaders/followers
- Proxy for strength of customer capital using expenditure share by older households in industry >
- Project difference in innovation between leaders and followers on proxy >

Industry panel

- Data:
 - ▶ Panel of industries, 1990 to 2019
 - ▶ R&D from Compustat, consumption share from Consumer Expenditure Survey
 - ▶ Restrict to industries with high percentage of output used as final goods
 - ▶ Take average observations in bins of 3 years

Age composition of demand

- Comovements of older households expenditure share with R&D spending difference between leader and follower ex
- Regressions:

$$Y_{jt} = \gamma S_{jt} + \eta A_{jt} + \delta_j + \nu_t + \varepsilon_{jt}$$

- ▶ Y_{jt} : Difference in R&D spending of top 90th productive firms and other firms in industry, standardized
- ▶ S_{jt} : Share of expenditures by households age 35 and over
- ▶ A_{jt} : Controls: Standard deviation of log revenue productivity; Total household expenditure on industry

Age composition of demand

| Dep var | $R\&D_{jt}$ | $\log(1 + R\&D)_{jt}$ |
|---------------------|-----------------|-----------------------|
| S_{jt} | 10.07 (1.72) | 11.96 (2.36) |
| N ind | 28 | 28 |
| N ind \times time | 224 | 224 |

T-stat in parentheses. Heteroskedastic robust standard errors.

[back](#)[sum](#)[more](#)[disp](#)[rsize](#)[weighted](#)[patents](#)

Age composition of demand

- Larger share of expenditure from older households:
 - ▶ Larger difference in innovation between top and non-top firms
 - ▶ Consistent with model
- Can compare to regression on model simulated data >

Quantifying effect of aging demographics

Aging demographics - Along the transition

- Transition along observed and predicted path of fraction of older households from 1960 to 2060, starting from BGP Fig bgp

| Year | Model | | Data |
|---------------------------------|--------|------------|------------------|
| | 1980 | 2020 | 1980-2020 change |
| R&D divergence | 0.0178 | +0.151 std | +0.524 std |
| Revenue productivity dispersion | 0.203 | +0.01 | +0.113 |
| Aggregate markups | 1.28 | +0.02 | +0.11 |
| Mean market share | 0.264 | +0.017 | +0.05 |
| Entry/Exit rate | 1.86% | -0.51% | -0.51% |

Aging demographics - Along the transition

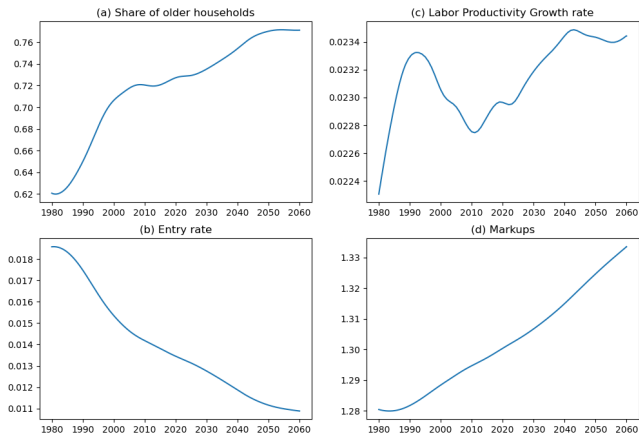


Figure: Evolution of measures along the transition

Effect of aging demographics

- More weight on older households in demand \rightarrow stronger effect of customer capital for firms
 - ▶ Leaders enjoy larger pool of customers with strong habits to sell to \sim more k for leaders
 - ▶ Followers sell less as pool of customers without habits for leader's good shrink \sim less k for followers
 - ▶ \Rightarrow Larger difference in innovation \Rightarrow Leaders widen productivity gap over followers
 - ▶ \Rightarrow Increased concentration, increased markups

Effect of aging demographics

- Aging demographics can account for sizable portion of trends in revenue productivity dispersion, aggregate markups, concentration
- ▶ Over the transition, changes are around 10%-35% of observed trends; predicted to keep increasing ➤

Innovation subsidies with customer capital

Subsidy to R&D

| 10% subsidy to R&D cost | With customer capital | Without customer capital |
|---------------------------------|-----------------------|--------------------------|
| Revenue productivity dispersion | +9.38% | +1.66% |
| Mean market share | +2.84% | +1.11% |
| Aggregate Markups | +1.32% | +0.39% |
| Entry rate | -5.55% | +0.41% |
| Growth rate | +8.64% | +8.62% |
| Welfare (CE) | +4.24% | +4.15% |

*Percentage deviation from baseline

Subsidy to R&D

- Leader and follower innovation increase proportionally \Rightarrow innovation difference increase \Rightarrow widen productivity gap
- With customer capital, leader produce more \Rightarrow build more customer capital \Rightarrow innovate more \Rightarrow further widen productivity gap entry

Conclusion

- Customer capital affects firm innovation and industry concentration as consequence
- Changes in customer capital, associated with aging demographics, generates sizable portion of aggregate trends in productivity dispersion, concentration, markups
- Effect of innovation subsidies on market structure amplified with customer capital
 - ▶ Additional consideration for policy makers when designing policies

Proxy

- Expenditure share by older households proxy:
 - ▶ Consumption significantly more persistent for households age 35 and older (Bornstein 2021) ~ larger customer capital effect
 - ▶ Proxy for strength of customer capital at the industry level [back](#)

Households - Habits

- Habits evolution [back](#)

$$k_{ijt+1} = (1 - \delta) \frac{0.5\epsilon^Y M^Y + k_{ijt} M^O (1 - \epsilon^O)}{\epsilon^Y M^Y + M^O (1 - \epsilon^O)} + \delta \left[\frac{p_{ijt} C_{ijt}^Y}{p_{ijt} C_{ijt}^Y + p_{-ijt} C_{-ijt}^Y} \epsilon^Y M^Y + \frac{p_{ijt} C_{ijt}^O}{p_{ijt} C_{ijt}^O + p_{-ijt} C_{-ijt}^O} M^O (1 - \epsilon^O) \right] \frac{1}{\epsilon^Y M^Y + M^O (1 - \epsilon^O)}$$

- External habits, accumulate from past expenditure of other old households
 - ▶ Average of young households turning old tomorrow and old households alive tomorrow
 - ▶ Representative old household consume more of good today → like it more → consume more tomorrow with less consideration for prices
 - ▶ Customer capital for the firms

Quant. model comparison

- Simulate model along transition path of ϵ^O to match fraction of older households from 1960 to 2060
- Run regression on simulated sectors from 1990 to 2019:
 - ▶ Project R&D spending difference between high and low productivity firms in sector, standardized, on share of expenditure by older households [back](#)

Quant. model comparison

| | Simulated | Empirical | |
|-------------------------|-----------|-------------------------|-------------------------|
| R&D | 6.60 | 11.89 (−0.52, 24.30) | 10.07 (−1.45, 21.59) |
| $\log(1 + \text{R\&D})$ | 6.62 | 10.41 (3.34, 17.62) | 11.96 (1.97, 21.95) |
| FE | Ind | Ind | Ind, Time |

95% confidence interval in parentheses [back](#)

Aging demographics - Comparing BGPs

- Decrease ϵ^O to match fraction of older households in 2020 [back](#)

| Fraction of older households | Model | | Data |
|---------------------------------|--------|------------|------------|
| | 0.65 | 0.72 | |
| R&D divergence | 0.0171 | +0.115 std | +0.524 std |
| Revenue productivity dispersion | 0.203 | +0.053 | +0.113 |
| Aggregate markups | 1.281 | +0.074 | +0.11 |
| Mean market share | 0.265 | +0.032 | +0.05 |
| Entry/Exit rate | 1.87% | -0.47% | -0.51% |
| Growth rate | 2.22% | +0.04% | -0.36% |

Disciplining habit parameters - Implementation

- Initial state A :
 - ▶ Same productivity across 2 firms; set $k_A^x > 0.5$ at long run level; calculate share S_A^x
- Change market condition:
 - ▶ Firm x innovates; with $k^x = k_A^x$, obtain new price under eqm policy rules
 - ▶ Hold price and productivity constant, get new long run shares S_B^x and track evolution of shares S_{it}^x
- Calibrate strength of habits so that $\frac{S_A^x - S_{i1}^x}{S_A^x - S_B^x}$ matches target [back](#)

Policy motivation

- Equilibrium inefficient:
 - ▶ Low production: Firm charge markups
 - ▶ Low innovation: Firm profit gains $<$ Social gains
- Government can improve on equilibrium through mix of production subsidy and innovation subsidy
- Consider subsidy to entry and incumbent R&D
 - ▶ Compare to BGP without customer capital >

Firms

- Define: $m = (\log q - \log q_-) / \log \lambda$; $\pi = p * Y - I$; \mathcal{R} indicator if firm is replaced by entrant
- Duopolist solve

$$v(k, k_-, m) = \max_{l, \iota} \pi(l, l_-, k, k_-, m) - \frac{\gamma}{2} \left(\log \left(\frac{1}{1 - \iota} \right) \right)^2 \\ + \beta E_{m', \mathcal{R}} [v(k', k'_-, m') (1 - \mathcal{R})]$$

- ▶ Choice of l affects π today and k' tomorrow
- ▶ Choice of ι affects m', \mathcal{R} tomorrow

eqm back

Subsidy to entry

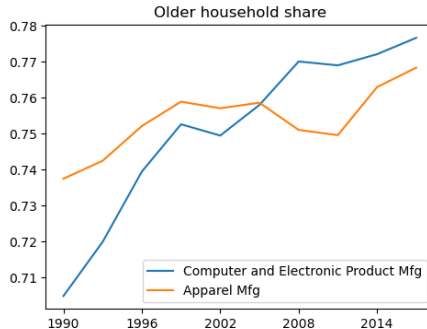
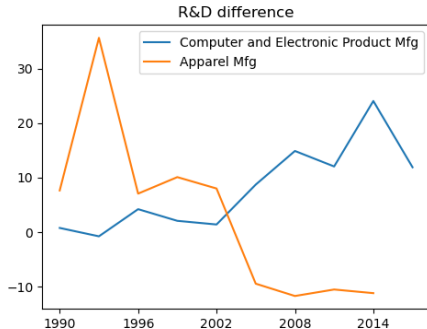
| 10% subsidy to entry cost | With customer capital | Without customer capital |
|---------------------------------|-----------------------|--------------------------|
| Revenue productivity dispersion | +0.71% | -1.09% |
| Mean market share | -1.04% | -0.45% |
| Aggregate Markups | -0.46% | -0.21% |
| Entry rate | +8.20% | +6.52% |
| Growth rate | +0.08% | +0.21% |
| Welfare (CE) | +0.13% | +0.12% |

*Percentage deviation from baseline

Subsidy to entry

- Entry subsidy decrease productivity dispersion, concentration, markups rnd
 - ▶ Entrant innovate on follower's tech. \Rightarrow higher entry reduce productivity gap
 - ▶ Larger effect with customer capital: Lower productivity gap \Rightarrow lower k for leaders \Rightarrow lower innovation difference \Rightarrow lower productivity gap

Example of 2 industries



[back](#)

Age composition of demand - scaled

| Dep var | $(R\&D / Emp)$ | $\log(1 + R\&D) / Emp$ | $(R\&D / Asset)$ | $\log(1 + R\&D) / Asset$ | $(R\&D / Sale)$ | $\log(1 + R\&D) / Sale$ |
|------------|----------------|------------------------|------------------|--------------------------|-----------------|-------------------------|
| <i>S</i> | 9.62 (2.57) | 6.44 (1.71) | 7.07 (1.61) | 6.74 (1.50) | 8.45 (1.61) | 8.67 (1.63) |
| N ind | 28 | 28 | 28 | 28 | 28 | 28 |
| N ind×time | 221 | 221 | 224 | 224 | 224 | 224 |

T-stat in parentheses. Heteroskedastic robust standard errors. [back](#)

Age composition of demand - weighted

| Dep var | $R\&D_{jt}$ | $\log(1 + R\&D)_{jt}$ |
|---------------------|----------------|-----------------------|
| S_{jt} | 4.86 (0.80) | 14.51 (2.20) |
| N ind | 28 | 28 |
| N ind \times time | 224 | 224 |

T-stat in parentheses. Heteroskedastic robust standard errors.

[back](#)

Age composition of demand - more

| Dep var | Top 90 th | | Bottom 90 th | |
|------------|----------------------|-----------------------|-------------------------|-----------------------|
| | $R\&D_{jt}$ | $\log(1 + R\&D)_{jt}$ | $R\&D_{jt}$ | $\log(1 + R\&D)_{jt}$ |
| S_{jt} | 7.96 (1.74) | 10.49 (2.89) | -0.38 (-0.16) | -1.41 (-0.62) |
| N ind | 28 | 28 | 28 | 28 |
| N ind×time | 232 | 232 | 265 | 265 |

T-stat in parentheses. Heteroskedastic robust standard errors. [back](#)

Age composition of demand - patents

| Dep var | $\log(1 + CW)$ | $\log(1 + CW) / Emp$ | $\log(1 + CW) / Asset$ | $\log(1 + CW) / Sale$ |
|------------|------------------|----------------------|------------------------|-----------------------|
| S_{jt} | -3.07 (-0.59) | 10.53 (1.72) | 15.03 (1.66) | 14.67 (1.73) |
| N ind | 28 | 28 | 28 | 28 |
| N ind×time | 235 | 235 | 235 | 235 |

CW: Citation weighted patent count, calculated for each firm i in time t as $\sum_{p \in P_{it}} 1 + \frac{C_p}{C_t}$. T-stat in parentheses.

Heteroskedastic robust standard errors. [back](#)

Effect of consumption shares on dispersion

- Changes in dispersion affected by gap in innovation rate between leader and follower

$$\Delta Disp_{jt+1} = (\iota_{ijt} - \iota_{-ijt}) \ln \lambda$$

- Regression for dispersion:

$$\Delta Disp_{jt+1} = \beta S_{jt} + \theta D_{jt} + \alpha_j + \eta_t + \epsilon_{jt}$$

- $\Delta Disp_{jt+1}$: Change in the standard deviation of log revenue productivity
- S_{jt} : share of expenditures by households age 35 and over
- D_{jt} : Controls: Total household expenditure on industry

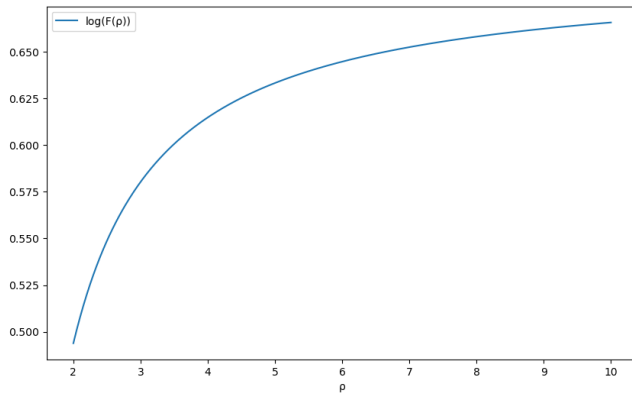
Effect of consumption shares on dispersion

| Dep var | $\Delta Disp_{jt+1}$ |
|---------------------|----------------------|
| S_{jt} | 0.75 (2.59) |
| N Ind | 28 |
| N Ind \times Time | 258 |

back

Log $F(\rho)$

back



Recursive Equilibrium

- Household policies, firm policies, firm value, and law of motion where
 - ▶ Household demand is optimal, given firm policies
 - ▶ Given household demand and competitor's policies, firm value solves the firm's Bellman and policies are consistent with maximization
 - ▶ Law of motion consistent with firm policies [back](#)

Households

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- Budget:

$$P_t^a C_t^a + P_t^A A_{t+1}^a = L_t^a + (P_t^A + d_t) A_t^a$$

- Aggregator: $C_t^a = \exp \left[\int \ln C_{jt}^a dj \right]$

$$C_{jt}^Y = \left(0.5^{\frac{-\theta}{\rho}} \left[0.5^{\frac{\theta}{\rho}} (C_{1jt}^Y)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} (C_{2jt}^Y)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \int^{\mathcal{N}} C_{fjt}^Y(x)^{\frac{\rho-1}{\rho}} dx \right] \right)^{\frac{\rho}{\rho-1}}$$

$$C_{jt}^O = \left(0.5^{\frac{-\theta}{\rho}} \left[k_{1jt}^{\frac{\theta}{\rho}} (C_{1jt}^O)^{\frac{\rho-1}{\rho}} + k_{2jt}^{\frac{\theta}{\rho}} (C_{2jt}^O)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \int^{\mathcal{N}} C_{fjt}^O(x)^{\frac{\rho-1}{\rho}} dx \right] \right)^{\frac{\rho}{\rho-1}}$$

Dispersion trend

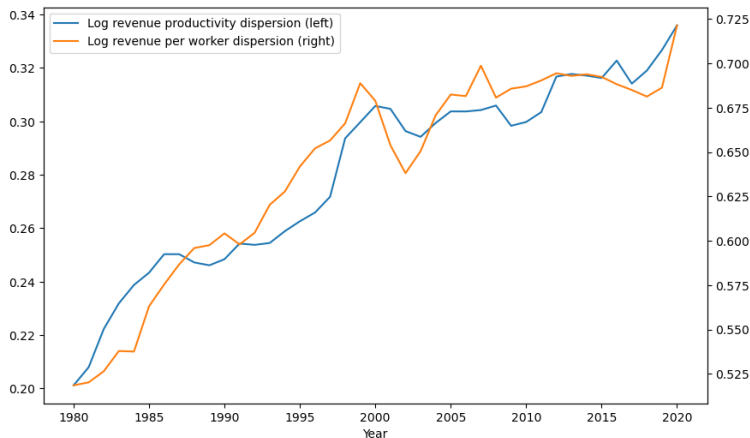
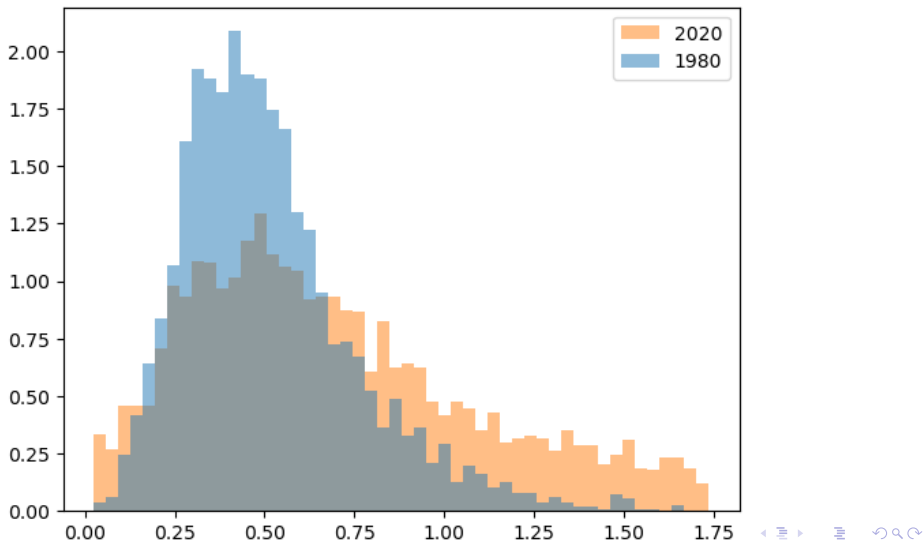


Figure: Between firm TFPR std and Sale/employment std [back](#)

Dispersion cross-section



R&D divergence

- Increasing divergence in R&D investment between more productive firms and less productive firms within industry

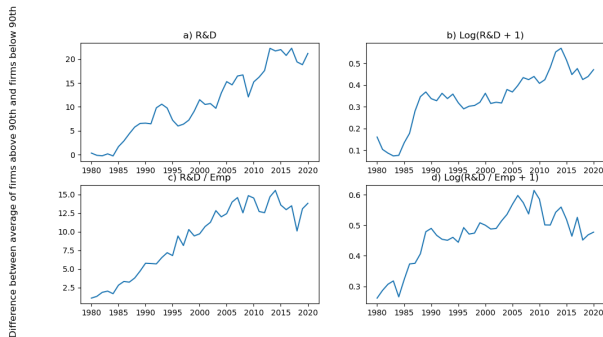


Figure: Difference of mean R&D spending between upper and lower firm quantiles by revenue

Age expenditure trend

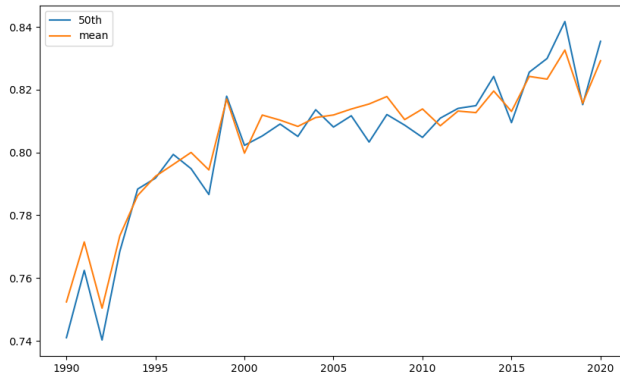


Figure: Share of expenditure of 36yo and above households, 3 digit NAICS [back](#)

Firms

- Firm profits: $\pi_{ijt} = p_{ijt} C_{ijt} - \frac{C_{ijt}}{q_{ijt}} \equiv s_{ijt} - l_{ijt}$, where s_{ijt} is implicitly defined by

$$s_{ijt} = \frac{p_{ijt}^{1-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx} M_y + \frac{(2k_{ijt})^\theta p_{-ijt}^{1-\rho}}{(2k_{ijt})^\theta p_{-ijt}^{1-\rho} + (2k_{-ijt})^\theta p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx} M_o$$
$$\frac{p_{-ijt}}{p_{ijt}} = \frac{l_{ijt}}{l_{-ijt}} \frac{s_{-ijt}}{s_{ijt}} \frac{q_{ijt}}{q_{-ijt}}; \quad \frac{p_{ijt}}{p_{fjt}} = \left(\frac{1}{q_{fjt}} \frac{\rho}{\rho - 1} \right)^{-1} \frac{s_{ijt} l_{ijt}^{-1}}{q_{ijt}}$$

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Discrete choice demand setup

- Sectoral preference:

$$C_{jt}^Y = \max_{ijt} \left[\exp \left(\frac{1}{\rho - 1} \epsilon_{ijt} \right) C_{ijt}^a \right]$$

$$C_{jt}^O (\{k_{ijt}\}) = \max_{ijt} \left[\exp \left(\frac{1}{\rho - 1} [\epsilon_{ijt} + \theta \ln (2k_{ijt})] \right) C_{ijt}^a \right]$$

Discrete choice demand setup

- Good chosen to solve:

- ▶ For young:

$$\max_{ijt} - (\rho - 1) \ln p_{ijt} + \epsilon_{ijt}$$

- ▶ For old:

$$\max_{ijt} - (\rho - 1) \ln p_{ijt} + \epsilon_{ijt} + \theta \ln (2k_{ijt})$$

with ϵ_{ijt} iid Type I Extreme Value

back

Summary stats

| | Difference | | |
|--------------------------------------|------------|-------|----------------|
| | S | R&D | $\log(1+R\&D)$ |
| Std, controlling for ind and time | 0.014 | 21.97 | 0.83 |

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Simple model - Discrete choice

- Unit mass households, 1 unit of endowment to spend [back](#)
 - ▶ Preference: $\exp\left(\frac{1}{\rho-1} [\epsilon_i^h + \theta \ln(k_i)]\right) c_i$ for $i \in \{1, 2\}$ with ϵ_i^h iid extreme value shocks
 - ★ Habits/Customer capital (k_1, k_2) ; Habit strength θ
 - ▶ Household choice: $i^h = \arg \max_i (1 - \rho) \log p_i + \theta \log k_i + \epsilon_i^h$
 - ▶ Choice probability of choosing i for household h :

$$\frac{k_i^\theta p_i^{1-\rho}}{k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}}$$

Eqm Profits

$$\pi(k_i/k_{-i}, q_i/q_{-i}) = \frac{\left(\frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}} \left(\frac{q_i}{q_{-i}} \right)^{(\rho-1)/\rho} + \frac{1}{\rho} \right) \frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}} \left(\frac{q_i}{q_{-i}} \right)^{(\rho-1)/\rho}}{\left[1 + \frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}} \left(\frac{q_i}{q_{-i}} \right)^{(\rho-1)/\rho} \right]^2}$$

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Relevance of Customer Capital

- Brand capital 6-25% of firm value (Belo et. al. 2022)
- Product familiarity associated with lower default risk (Larkin 2013)
- Differences in customer base accounts for 80% sale variances (Einav et. al. 2021, Afrouzi et. al. 2023)
- Firm spending on advertising, sales expenditures, customer service around 2/3 of physical capital spending (He et. al. 2024)
- Firms stabilize prices to maintain long-run customer relationship (Blinder et. al. 1998, Fabiani et. al. 2006)
- New firm formation declines when consumer inertia rises (Bornstein 2021) [back](#)