Customer capital and firm innovation*

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Abstract

This paper studies the role of customer capital in driving firm innovation decisions and the resulting effects on aggregate productivity and concentration. I develop a step-by-step innovation model where households form deep habits in consumption. These habits form customer capital for firms: firms can decrease prices and increase production to build customer capital and raise future profits, at a potential loss to current profits. As the strength of habits increase, leader firms face higher and more inelastic demand while followers face lower demand. I show how these movements in demand result in an increase in innovation by leader firms relative to follower firms, leading to greater productivity dispersion and concentration. I find evidence for this effect in data on U.S. public firms: in sectors where outputs are more heavily consumed by older households—those with stronger habits—the most productive firms increase their R&D investment relative to others. I discipline the strength of habits in the model base on micro estimates of household evolution of consumption. I then use the model to quantify the effects of changes in aggregate customer capital arising from aging demographics. The model suggests that the shift toward older households between 1980 and 2019 accounts for 10%-35% of the observed trends in rising revenue productivity dispersion among firms, increasing market concentration, and higher aggregate markups. The model also highlights how customer capital influences the effectiveness of innovation policies: with customer capital, innovation subsidies have a significantly larger impact on concentration and markups—around two to three times greater than in an environment without customer capital.

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1 Introduction

Customer capital constitutes an important form of intangible capital for the firm. Differences in customer base account for approximately 80% of variance in sales across firms (Einav et al. 2021, Afrouzi et al. 2023). It contributes to increased brand familiarity and improved brand perception - a substantial portion of firm value[1]. Higher brand familiarity is associated with lower cash flow volatility and lower default risk (Larkin 2013). Empirical evidence documents that the desire to acquire and maintain customer capital drives firm decisions. Firms spend considerable amounts on advertising and sales expenses. Firms keep price stable despite changing production costs in order to maintain long-run customer relationships (Blinder et al. 1998, Fabiani et al. 2006). Differences in consumption persistence across states also drives differences in new firm formation (Bornstein 2021).

In this paper, I build a model to study how customer capital drives firm incentives to innovate and the resulting consequences on aggregate productivity, concentration, and markups. Using the model, I quantify how aging demographics, by varying the effect of customer capital, increases Research and Development (R&D) differences between firms, concentration, and markups. I also show how customer capital and its impact on innovation matters for the outcomes of government innovation policies.

I develop a step-by-step model of innovation that incorporates consumption habits. Consumption habits form customer capital for firms, whereby when a firm lowers their price and increases their sales in the current period, they enjoy higher and more inelastic demand in future periods. Changes in future demand affects firm incentives to do R&D and innovate, which then affects productivity dispersion and concentration. I provide empirical support for the model, proxying for the strength of customer capital using variations in the age composition of demand within industries. I then use the model to quantify the effect of aging demographics on innovation and productivity dispersion. The induced rise in the share of older households in aggregate demand can account for 10%-35% of the increase in divergence of R&D spending across firms, the increase in revenue productivity across firms, and the rise in aggregate markups and industry concentration. Finally, I consider the impact of government innovation subsidies, in the model and in an environment without customer capital as comparison. I find that with customer capital, innovation subsidies have larger impacts on productivity dispersion, markups, and industry concentration - two to three times as much compared to an environment without customer capital.

My model builds on the step-by-step innovation framework of Aghion et al. (2001). There are many industries in the economy, and in each industry two dominant firms compete. The

[1]Belo et al. (2022) estimates brand capital to be 6-25% of firm market value

two firms engage in Cournot competition, choosing the quantities of their products to supply. Each firm can invest in R&D to reduce their future production costs, in order to have lower prices and capture more of the market. I add to this framework consumption habit formation (Ravn et al. 2006), whereby the larger a household's current expenditure on a good, the stronger their future preference for the good. Alongside the market structure, consumption habits imply that if a firm increases sales in a given period, demand for their product in future periods will be higher and more inelastic. Firms can invest in their customer capital stock by increasing supply and lowering price, which reduces current profits but allows for higher profits in the future.

To develop intuition for the main results, I begin with a simple static model to understand how the increase in demand and decrease in demand elasticity arising from larger customer capital stock affect firm innovation. The simple model treats customer capital as exogenous but allows for a sharp characterization of the two demand effects. Higher demand incentivizes the firm to raise quantity supplied, which increases the incentive to innovate to reduce production cost. Lower demand elasticity incentivizes the firm to restrict supply in favor of charging higher markups, which decreases the incentive to innovate. I establish a condition on the relative revenue productivity of the firm that determines which effect dominates. When the firm's revenue productivity relative to its rival is below a threshold, more customer capital leads to the firm increasing innovation.

The full model retains features of demand from the simple model, and endogenizes the evolution of customer capital and productivity. On the household side, there are two types of households, young and old, that differ in the strength of their consumption habits. This heterogeneity propagates the effect of aging demographics. Specifically, I assume that only old households form habits, thus loading all customer capital effects on old households. Habits are exogenous to any individual household and are determined by past expenditures by the average old household in the economy. On the firm side, in each industry, in addition to the two dominant firms, there is a mass of fringe firms. The fringe firms do not innovate nor build customer capital, and they charge constant markups. The addition of fringe firms generates flexibility for the model to match empirical levels of industry concentration and markups. The number of dominant firms is fixed at two, though these firms still face the threat of being replaced by entrants. Specifically, in each industry each period, a potential entrant conducts R&D. They enter if they successfully innovate, replacing the lower productivity firm and inheriting their stock of customer capital.

A key parameter in the model is the strength of consumption habits. I discipline this parameter base on empirical work by Bronnenberg et al. (2012), which documents how migrants' consumption patterns evolve as they move from one market to another. The strength of habits is inferred from how little a migrant's consumption changes in the period after they move, relative to the average consumption difference between the destination market and the origin market. I conduct an analogous exercise in the model by tracking how market shares evolve following a change in product prices. The key moment is how much market shares move in the period after the prices change relative to how much market shares move after a long period of time. Strong habits imply small movements in market shares for the immediate period following the price change, as households stick to their consumption patterns regardless of variations in prices.

I turn to the data for empirical support for the calibrated model. I conduct the analysis at the industry level, proxying for the strength of customer capital in the industry using the expenditure share by older households[2]. I construct an industry panel, with a measure of older consumer shares in an industry from the Consumer Expenditure Survey, and industry R&D difference and productivity dispersion from public firms. By projecting R&D difference and change in productivity dispersion onto the older consumer share, I find that a higher expenditure share from older consumers is associated with larger R&D differences and more positive changes in dispersion. This is in line with predictions of the model. I also run similar projections on simulated data from the calibrated model, obtaining coefficients that are comparable to the empirical counterparts.

The link between the expenditure share and innovation is as follows. When the expenditure share of older consumers within an industry is high, it implies increased demand and decreased elasticity for more productive firms, as they can sell to older consumers who over time build habits for their products. Less productive firms face decreased demand, as much of their demand is from younger consumers who have not yet built habits. A rise in the expenditure share of older consumers is akin to an increase in customer capital for the more productive firms, and a decrease in customer capital for the less productive firms. Given the amount of revenue productivity dispersion in the data, we would expect a higher expenditure share to increase divergence in innovation between high and low productivity firms, and with it, increase productivity dispersion.

I use the model to perform two exercises. The first is to quantify the effect of changes in aggregate customer capital induced by aging demographics. Aging demographics increases the share of old households in the economy, which increases the strength of customer capital in the aggregate. I evaluate this in two ways, a comparison of balanced growth paths (BGPs), and changes along the transition from one BGP to another. For the BGP comparison, I start with a baseline economy calibrated to the US in 1980, and compare it to an economy where

[2] Bornstein (2021) provides evidence that households above 35 years hold have significantly higher consumption persistence. This can be interpreted as older households having stronger customer capital effects, whereby past consumption matters more for current consumption, conditional on price and quality.

the share of old households takes the value of that in 2020. The larger old household share induces more productive firms to increase innovation compared to their competitors. This results in higher productivity dispersion, higher concentration, and higher aggregate markups, around 45%, 65%, and 65%, respectively, of the increases observed in the data over 1980 to 2020. For the transition, I assume that the economy starts from a BGP in 1960. I feed in the path of the share of old households forecasted until 2060, and calculate the transition to the new BGP corresponding to the 2060 share of old households. Effects along the transition are quantitatively smaller than the BGP comparison due to the slow transition. From 1980 to 2020, the model generates increases in productivity dispersion, concentration, and aggregate markups that is around 10%, 35%, and 20%, respectively, of the increases observed in the data over this period.

In the second exercise, I use the model to analyze how the inclusion of customer capital affects the outcome of government innovation policies. I consider two policies, a subsidy to entry and a subsidy to R&D. Firms in the model underinvest in innovation compared to the social optimum, motivating the desire for such policies. Moreover, entry subsidies and R&D subsidies are widely used in practice. In the model, these two amount to decreasing the cost of innovation for potential entrants and incumbent firms. They have some opposing outcomes: the subsidy to entry decreases productivity dispersion, concentration, and markups, and increases entry, while the subsidy to R&D increases productivity dispersion, concentration, and markups, and decreases entry. Compared to an environment without customer capital, the effect of these two policies on productivity dispersion, concentration, and markups is around two to three times as large. This arises from the interaction between customer capital and firm innovation decisions. For example, consider the subsidy to R&D, which increases innovation for both the leader - the more productive firm - and follower proportionally. This raises the absolute difference in the innovation rates, so that over time the gap in productivity widens and leaders become even more productive compared to their competitors. Leaders then are able to capture more market share and build up more customer capital. Higher customer capital for the leaders in return increases their incentive to innovate, further widening the productivity gap. The feedback effect amplifies the impact of innovation policies on the market structure. Policies aimed at promoting innovation could significantly affect concentration as an unintended outcome, generating additional considerations for policy makers when designing policy.

Related Literature

This paper contributes to literature on the effect of customer capital on firm decisions and outcomes. Theoretically, industrial organization papers have studied how customer capital, arising through lock-ins and switching costs, affects market competitiveness. Beggs and Klemperer (1992) show how lock-ins result in decreased competitiveness, increased price markups and profits. Dube et al. (2009) find that switching costs could instead increase competitiveness, depending on the size of the switching costs. In the macroeconomic context, Ravn et al. (2006) study how the introduction of consumption habits affect firm cyclical price setting in a dynamic stochastic general equilibrium model. Gourio and Rudanko (2014) study customer capital that arises from frictional matching between consumers and producers, and its effect on firm investment, sales, and markups. Empirically, Larkin (2013) provides evidence on the effect of customer brand perception on firm finances, investments, and defaults. Baker et al. (2023) finds that customer churn is predictive of firm valuation. Afrouzi et al. (2023) and Einav et al. (2021) document that differences in customer base among firms can account for 80% of the variance in firm sales. This paper contributes to this literature by studying the effect of customer capital on firm innovation.

Customer capital is a form of intangible capital for firms, and recent papers have explored how other forms of intangibles can affect innovation. On the supply side, with intangible capital that contributes to firm productivity, De Ridder (2024) studies how firm heterogeneity intangible efficiency generates heterogeneity in innovation. On the demand side, with intangible capital that alters firm demand, Shen (2023), Cavenaile and Roldan-Blanco (2021) and Cavenaile et al. (2024) explore how advertising interacts with firm innovation decisions. Unlike advertising in these papers, customer capital is a persistent object for firms that evolves dynamically and is dependent upon past sales. The treatment of customer capital in my paper is most similar to Ignaszak and Sedlacek (2023), where firms need to build demand as they innovate and grow. While these authors study the behavior of monopolistically competitive firms, I consider a market structure where firms are large relative to their market and compete directly with each other. The role of competition in innovation is important, as studied in Aghion et al. (2005), and customer capital changes firms incentives to compete as well as their incentives to innovate. Moreover, my modeling of the market structure allows a clean connection between the effects of customer capital on firm innovation and changes in market concentration.

Other papers have studied the effects of aging demographic on firms. Hopenhayn et al. (2022) and Karahan et al. (2019) consider the effects of reduced labor supply growth due to lower population growth. They find lower rates of new firm formation and older firms in the economy on average as labor supply growth declines. Peters and Walsh (2021) further this argument in the context of productivity growth, where the decline in new firm formation results in lower creative destruction and innovation, leading to a decline in the aggregate productivity growth rate. Bornstein (2021) considers the effect of demographic shift on firm demand through the customer capital channel. Unlike Bornstein (2021), which focuses on firm pricing decisions, this paper emphasizes the effect of customer capital on firm innovation.

Recent papers have linked trends in concentration and productivity using models of endogenous growth, with several papers applying the step-by-step innovation framework of Aghion et al. (2001) as in this paper. Akcigit and Ates (2023) explore the effect of lower rate of knowledge diffusion from leaders to followers. Liu et al. (2022) study the impact of lower firm discounting, due to falling interest rates. Olmstead-Rumsey (2022) considers the consequences of declining innovative efficiency from followers. These papers generate a similar result, where innovation efforts by the leaders increase relative to followers, which then drive the increase in concentration. My paper also shares this result. Aside from linking aggregate trends, the inclusion of customer capital into the step-by-step innovation framework has further implications for government innovation policies.

2 Simple model of customer capital and innovation

I first consider a static industry equilibrium to understand how changes in customer capital affect firm innovation decisions. There are two firms in the industry that differ in productivity. They can make R&D investments in order to increase their productivity and obtain higher profits. I model customer capital as predetermined variables that alter consumer demands for firm products.

The static model highlights the mechanism underlying the quantitative model in section 3, which share similarities in setup. Customer capital have competing effects on firm innovation, and the static model gives a sharp characterization on which effect dominates. More customer capital both increase demand and reduce demand elasticity. If the firm increases supply to meet higher demand, they would also increase innovation to reduce production cost. The firm could instead decrease supply and charge higher markups as demand is less elastic, which decreases the incentive to innovate. Which effect dominates depends on how large the revenue productivity of the firm is compared to its rival.

2.1 Setup

Consider an industry with two firms producing differentiated goods. Households can consume a mix of the goods, and their preferences for each good are affected by a term I refer to as customer capital. The firms differ in productivity, and can invest in R&D to increase their productivity probabilistically.

Households are unit mass, with each household endowed with 1 unit to spend on goods in the industry. Their utility from consuming a mix (c_1, c_2) is given by

$$U(c_1, c_2) = \left(k_1^{\frac{\theta}{\rho}} c_1^{\frac{\rho-1}{\rho}} + k_2^{\frac{\theta}{\rho}} c_2^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}.$$

Here (k_1, k_2) are predetermined customer capital for good 1 and 2 respectively, $\rho > 1$ determines the substitutability between the two goods, and $\theta \geq 0$ determines how much customer capital matter for demand. Customer capital affects preferences for the corresponding good, where a large k_i for good $i \in \{1, 2\}$ can be interpreted as households liking good i a lot. Given these preferences, inverse demand for good i can be derived as

$$p_i = \frac{(k_i)^{\theta/\rho} c_i^{-1/\rho}}{(k_i)^{\theta/\rho} c_i^{\frac{\rho-1}{\rho}} + (k_{-i})^{\theta/\rho} c_{-i}^{\frac{\rho-1}{\rho}}}, \tag{1}$$

where $-i$ denotes the rival good to i.

For the firms, the timing is that firms invest in R&D, the outcomes of these investments are realized, then production occurs. It is more convenient for exposition to consider two periods. In the first period, firms make R&D investment decisions that affect their second period productivity. Production then occurs in the second period. I assume firms compete a la Cournot.

Firm i in period 1 has initial productivity $\mathring{q}_i > 0$. They can invest in R&D to increase their productivity in period 2 probabilistically by a factor of λ. Specifically, their productivity in period 2, denoted by q_i, is $\lambda \mathring{q}_i$ with probability ι_i, and $\mathring{q}_i$ with probability $1 - \iota_i$. I refer to the success probability ι_i as the innovation rate. The firm has to spend $\frac{\gamma}{2}\iota_i^2$ in R&D to achieve an innovation rate of ι_i. In period 2, firm i produces with constant marginal cost $\frac{1}{q_i}$.

Going backwards, given second period productivity (q_i, q_{-i}), second period payoffs can be solved for. First period innovation rates are then obtained from first order conditions. The following proposition characterizes the second period payoffs and first period innovation decisions. All derivations are relegated to Appendix B.

Proposition 1. *For the industry duopoly,*

a) Second period payoff for firm i,π_i, is given by

$$\pi_i = \pi\left(k_i/k_{-i}, q_i/q_{-i}\right) = \frac{\left(\frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}} \left(\frac{q_i}{q_{-i}}\right)^{(\rho-1)/\rho} + \frac{1}{\rho} \right) \frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}} \left(\frac{q_i}{q_{-i}}\right)^{(\rho-1)/\rho}}{\left[1 + \frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}} \left(\frac{q_i}{q_{-i}}\right)^{(\rho-1)/\rho}\right]^2};$$

b) For small (ι_i, ι_{-i}), innovation ι_i is approximated by

$$\iota_i = \frac{1}{\gamma}\left[\pi\left(k_i/k_{-i}, \lambda \mathring{q}_i/\mathring{q}_{-i}\right) - \pi\left(k_i/k_{-i}, \mathring{q}_i/\mathring{q}_{-i}\right)\right]. \tag{2}$$

Second period payoff is increasing in customer capital and productivity. The innovation choice ι_i is determined by the gains in second period payoff from increasing productivity by a proportion λ. The customer capital stocks (k_i, k_{-i}) affect these gain, hence affect innovation choice. The next subsection details what happens to innovation as customer capital changes.

2.2 Response of innovation to changes in customer capital

Consider the effect of changes in customer capital on innovation decisions. Movements in customer capital arise in a dynamic setting, where firms could influence their customer capital tomorrow through actions taken today. Specifically, the model in section 3 assumes that higher market share today generates higher customer capital tomorrow. I am interested in whether the higher customer capital would increase or decrease firm innovation incentives.

Define m_i such that $\frac{\mathring{q}_i}{\mathring{q}_{-i}} = \lambda^{m_i}$. m_i can be thought of as initial productivity step between firm i and its rival. Since productivity increases proportionally by λ upon innovation success, the productivity step can go up 1 if firm i succeeds, or down 1 if its rival $-i$ succeeds. Let $\kappa_i \equiv \left(\frac{k_i}{k_{-i}}\right)^\theta$ and redefine second period profits in terms of m_i, κ_i. The innovation decision in equation 2 can be rewritten as

$$\iota_i = \frac{1}{\gamma}\int_{m_i}^{m_i+1} \frac{\partial \pi\left(\kappa_i, x\right)}{\partial x} dx.$$

We can sign the response of ι_i to changes in customer capital by evaluating $\frac{\partial^2 \pi(\kappa_i, m_i)}{\partial m_i \partial \kappa_i}$. This is given in the following proposition.

Proposition 2. $\frac{\partial^2 \pi(\kappa_i, m_i)}{\partial m_i \partial \kappa_i} > 0$ *iff*

$$\kappa_i^{1/\rho}\lambda^{m_i(\rho-1)/\rho} < \sqrt{4\left(1-\frac{1}{\rho}\right)^2\left(2-\frac{1}{\rho}\right)^{-2} + \frac{1}{\rho}} + 2\left(1-\frac{1}{\rho}\right)\left(2-\frac{1}{\rho}\right)^{-1} \equiv F\left(\rho\right). \tag{3}$$

Proposition 2 implies that an increase in customer capital for firm i could either increase or decrease its innovation, depending on how large firm i's revenue productivity is relative to its competitor. The term on the left hand side is the ratio of revenue productivity, and the term on the right hand side gives the threshold. The right hand side term is a function only of the substitution parameter ρ, which I refer to as $F\left(\rho\right)$ for brevity. Figure 3 in Appendix

G graphs $F(\rho)$. Intuitively, innovation can go either way because an increase in customer capital both raises demand and reduces demand elasticity, resulting in competing effects on innovation incentives. Higher demand incentivizes the firm to raise production, which encourages innovation to reduce the cost of higher production. Lower elasticity incentivizes the firm instead to reduce production to increase markups, lowering the incentive to innovate.

To fix ideas, consider seperately the case of a follower - the firm with the lower initial productivity, and the case of a leader - the firm with the higher initial productivity. For firm i as the follower, with $m_i < 0$, further assume that the firm have lower customer capital compared to its rival, which is the empirically relevant case when customer capital reflects the firm's market share. Then we have $\kappa_i^{1/\rho}\lambda^{m_i(\rho-1)/\rho} < 1$. Since $F(\rho) > 1$ for $\rho > 1$, we also have $\kappa_i^{1/\rho}\lambda^{m_i(\rho-1)/\rho} < F(\rho)$. More customer capital for the follower raises its innovation. For firm i as the leader, with $m_i > 0$, also assume that the firm have higher customer capital compared to its rival. Now, whether $\kappa_i^{1/\rho}\lambda^{m_i(\rho-1)/\rho}$ is greater or smaller than $F(\rho)$ is ambiguous. More customer capital for the leader could either raise or lower its innovation, depending on the leader's relative revenue productivity.

I focus on the case when inequality 3 holds for the leader. I argue in subsection 4.3 that for the level of relative revenue productivity in the data, we would expect inequality 3 to be satisfied for the majority of cases.

Movements in customer capital affects revenue productivity dispersion in the industry. Within the industry, an increase in customer capital for firm i will increase κ_i as well as decrease κ_{-i} for its rival. Take firm i as the leader, and further assume that 3 holds. More customer capital for firm i increases its own innovation ι_i, and decreases its rival's innovation ι_{-i}. We can show that the expected change in the standard deviation of log productivity from period 1 to period 2 is proportional to the difference in innovation rates, $\iota_i - \iota_{-i}$. That is, expected productivity dispersion increases if the leader innovates more than the follower. Since higher customer capital for the leader increases the leader's innovation and decreases the follower's innovation, expected productivity dispersion increases. Expected revenue productivity dispersion increases, both from the higher κ_i and from the higher productivity dispersion.

We can use the static model to analyze the direction of effects arising from aging demographics, as in section 5. Under the assumption that older households have stronger customer capital effects (Bornstein 2021; Bronnenberg et al. 2012), aging demographics generates stronger overall customer capital effect for firms, as well as facilitates higher capital stocks as their consumers live longer. In the static model, this can interpreted as an increase in θ, which increases κ_i for the leader. The leader benefits from the larger pool of older households with strong customer capital effects, who over time grow attach their good. Whereas for the follower, their demand decreases as the pool of young, unattached households shrink. If we also have that firms are not too far apart in their revenue productivity, then aging demographics would cause leaders to innovate more compared to their rivals, increasing revenue productivity dispersion.

There are additional feedback effects once we consider dynamics that this static setup misses. Customer capital is built over time, accruing from past sales. A firm with higher sales today would see its customer capital increase in the future. The leader, being more productive and producing more than its rival, would accrue a higher customer capital stock over time. The higher customer capital stock in turn incentivize the leader to increase innovation, further widening the productivity gap with its rival.

This feedback effect is important in understanding how customer capital affect the outcomes of government innovation policies, as in section 6. If a policy changes the productivity gap between firms, it also changes firms' customer capital stock. This then affect firms' innovation decisions, in turn amplifying the effects of the policy on the productivity gap. For example, a proportional R&D subsidy would increase leader and follower innovation rates proportionally the same. It also leads to an increase in the absolute difference in innovation rates between the leader and follower, $\iota_i - \iota_{-i}$. This results in an increase in the productivity gap between the leader and follower. With a higher productivity gap, the leader production relative to the follower increases, generating larger customer capital stock for the leader over time. More customer capital then leads to higher innovation by the leader, amplifying the R&D subsidy's effect on the productivity gap.

3 A model of consumption habits and firm innovation

In this section, I develop a quantitative model of innovation with customer capital that arise through consumption habits. The model builds on the step-by-step innovation framework of Aghion et al. (2001). There is a continuum of industries in the economy, with two dominant firms in each industry engaging in a race in innovation to be the market leader. Additionally, firms accumulate consumption habits for their products (Ravn et al. 2006). These habits boost firm demand, as in the static model, and is built from past household consumption. I also allow for entry and exit of dominant firms.

Households are of two types, young and old, and they age and die stochastically. The two types differ in their strength of consumption habits: young households are not affected by habits, while old households are. This heterogeneity is important when addressing the effects of aging demographics in section 5.

The key difference between the quantitative model here and the static model in section

2 is dynamics. While any single period in the quantitative model shares similarities in market structure with the static model, customer capital and productivity in the quantitative model are endogenously determined. Firms have incentives to increase their production to build customer capital for future periods. Increasing production is done through increasing inputs as well as increasing productivity. So in addition to the effect of customer capital on innovation analyzed in the static model, firms also innovate in order to obtain more customer capital.

3.1 Environment

3.1.1 Households

There are two types of households in the economy, young and old, with a total mass of 1. Each period t, a random portion ϵ^Y of young households become old, a random portion ϵ^O of old households leave the economy, and new young households enter the economy. The mass of entering young household is the same as the mass of exiting old households. Denote the mass of young and old households by M^Y and M^O respectively. For other variables to be introduced, superscripts Y, O will be used to refer them to young and old households respectively.

Households consume a mix of goods produced in the economy, supply labor, and trade claims to a bundle of all firms in the economy. They have time separable utility and discount the future at rate β. Their per period prefences are given by

$$U_t^a = \ln C_t^a - L_t^a,$$

where $a \in \{Y, O\}$, C_t^a is a consumption aggregator over all goods, and L_t^a is the labor supply. Their budget constraint is

$$P_t^a C_t^a + P_t^A A_{t+1}^a = L_t^a + \left(P_t^A + d_t\right) A_t^a,$$

where A_t^a is a claim to a bundle of all firms in the economy, P_t^a is the price index on aggregated consumption, P_t^A is the price of the firm bundle, and d_t is the bundle's dividend payout. I normalize the wage to 1. The price index on aggregated consumption is allowed to differ by household types: while households face the same price for individual goods, differences in their habits will lead to different price indexes.

Goods are differentiated across and within sectors. There is an unit measure of sectors, denoted by j, with sectoral goods aggregated by $C_t^a = \exp\left[\int \ln C_{jt}^a dj\right]$. In each sector j, a pair of duopolists and a fringe of firm of mass $\mathcal{N}$ produce imperfectly substitutable goods.

The aggregated good for sector j is given by

$$C_{jt}^{Y}=\left(0.5^{\frac{-\theta}{\rho}}\left[0.5^{\frac{\theta}{\rho}}\left(C_{1jt}^{Y}\right)^{\frac{\rho-1}{\rho}}+0.5^{\frac{\theta}{\rho}}\left(C_{2jt}^{Y}\right)^{\frac{\rho-1}{\rho}}+0.5^{\frac{\theta}{\rho}}\int^{\mathcal{N}}C_{fjt}^{Y}\left(x\right)^{\frac{\rho-1}{\rho}}dx\right]\right)^{\frac{\rho}{\rho-1}}$$

$$C_{jt}^{O}=\left(0.5^{\frac{-\theta}{\rho}}\left[k_{1jt}^{\frac{\theta}{\rho}}\left(C_{1jt}^{O}\right)^{\frac{\rho-1}{\rho}}+k_{2jt}^{\frac{\theta}{\rho}}\left(C_{2jt}^{O}\right)^{\frac{\rho-1}{\rho}}+0.5^{\frac{\theta}{\rho}}\int^{\mathcal{N}}C_{fjt}^{O}\left(x\right)^{\frac{\rho-1}{\rho}}dx\right]\right)^{\frac{\rho}{\rho-1}}, \tag{4}$$

where ρ determines how substitutable goods within the sector are. Household, specifically old household, utility for consuming each firm's good is affected by their habit stock, with the strength of this effect governed by θ. Larger habit stock for a good, relative to other goods in the sector, has the interpretation that households like that particular good more.

Young households have equal habit stocks, constant at 0.5, across all goods. As such, consumption habits do not affect young households. Old households form habits, but only for the duopolist goods. I set their habit stocks for fringe goods constant at 0.5. Their habit stocks for duopolist goods, k_{1jt} and k_{2jt}, are determined based on past expenditure on these goods. I assume that these habits are external: they are determined by the average expenditure of other old households, so that a household's own consumption does not affect their habit stock. Moreover, they are common for all old households. Habit stock for duopolist good $i \in \{1,2\}$ evolves according to

$$\begin{aligned}k_{ijt+1}=&\frac{(1-\delta)}{\epsilon^{Y}M^{Y}+M^{O}\left(1-\epsilon^{O}\right)}\left(0.5M^{Y}\epsilon^{Y}+k_{ijt}M^{O}\left(1-\epsilon^{O}\right)\right)+ \\ &\frac{\delta}{\epsilon^{Y}M^{Y}+M^{O}\left(1-\epsilon^{O}\right)}\left(\frac{p_{ijt}C_{ijt}^{Y}}{p_{ijt}C_{ijt}^{O}+p_{-ijt}C_{-ijt}^{Y}}M^{Y}\epsilon^{Y}+\frac{p_{ijt}C_{ijt}^{O}}{p_{ijt}C_{ijt}^{O}+p_{-ijt}C_{-ijt}^{O}}M^{O}\left(1-\epsilon^{O}\right)\right).\end{aligned} \tag{5}$$

The first term represents the average habit stock for old households in period t, taken as the weighted average of habit stocks from young households that newly turned old -which takes a value of 0.5, and habit stocks of surviving old households - which takes a value of k_{ijt}. The second term represents expenditure on good i relative to total expenditure on good i and the rival duopolist good $-i$ for old households. Like the first term, this relative expenditure is the weighted average for young households newly turned old and surviving old households. δ determines the speed at which customer capital changes.

I assume that starting habit stocks at time $t=0$, k_{ij0}, k_{-ij0}, is such that $k_{ij0}+k_{-ij0}=1$ for all sectors j. The above evolution implies $k_{ijt}+k_{-ijt}=1\ \forall t,j$. $k_{ijt}>0.5$ corresponds to larger habit stocks for good i compared other goods, hence households have increased preference for good i. At $k_{ijt}=0.5$, all goods in sector j have the same level of habit stocks, and households do not have additional preference for good i. As such, $k_{ijt}=0.5$ represents a neutral level of habits, and is the rationale for setting the level of young households habit stocks.

Habits for households form customer capital for the duopolists. Firms understand that habit is a source to boost demand for their goods. And while external to households, the firms take into the evolution of habits. I refer to habit stock as customer capital when discussing firms.

Note that for firm i, $k_{ijt} > 0.5$ corresponds to the firm having larger customer capital stock compared to its rival. $k_{ijt} = 0.5$ represents a neutral level of customer capital, which I set the level of young household's stock to.

3.1.2 Firms

In each sector, firms engage in Cournot competition. Production technology is given by

$$Y_{ijt} = q_{ijt} l_{ijt},$$

where $Y_{ijt}, q_{ijt}, l_{ijt}$ is the firm's goods output, productivity, and labor input. Firm demand is given by summing demand across households:

$$C_{ijt} = \left(C^{Y}_{ijt} M^{Y} + C^{O}_{ijt} M^{O}\right).$$

Firms can invest in R&D to improve productivity probabilistically. To achieve a success probability of ι_{ijt}, the firm has to employ $\frac{\gamma}{2}\left(\log\left(\frac{1}{1-\iota_{ijt}}\right)\right)^2$ units of labor in R&D. Let D_{ijt} be an indicator function for R&D success. The firm's increase in productivity when $D_{ijt} = 1$ depends on whether the firm is a leader or a follower in the sector. I refer to the firm with the higher productivity, or higher customer capital in the case of equal productivity, as the leader, and their success indicator and productivity is denoted by $\overline{D}_{jt}$ and $\overline{q}_{jt}$. For the follower, their success indicator and productivity is denoted by $\underline{D}_{jt}$ and $\underline{q}_{jt}$.

For the leader, productivity increases proportionally by a factor $\lambda > 1$, so that

$$\overline{q}_{jt+1} = \overline{D}_{jt} \lambda \overline{q}_{jt} + \left(1 - \overline{D}_{jt}\right) \overline{q}_{jt}.$$

If the follower successfully innovates, they have an additional probability ϕ of achieving a breakthrough. The breakthrough innovation increases the follower's productivity to the level of the leader's. That is,

$$\underline{q}_{jt+1} = \underline{D}_{jt} \left(1 - \Phi\right) \lambda \underline{q}_{jt} + \underline{D}_{jt} \Phi \overline{q}_{jt} + \left(1 - \underline{D}_{jt}\right) \underline{q}_{jt},$$

where $\Phi = 1$ with probability ϕ.

Let m_{ijt} be such that $\lambda^{m_{ijt}} = \frac{q_{ijt}}{q_{-ijt}}$. m_{ijt} encodes the relative productivity between firm i and its rival in sector j, which I refer to as the technology gap. Assuming that $q_{ij0} = 1 \quad \forall i, j$, m_{ijt} is integer valued. Moreover, assume that there is a maximal gap $\bar{m}$ such that $\bar{m} \geq m_{ijt} \geq -\bar{m}$.

Fringe firms do not innovate. I assume that their technology level is a weighted geometric average of the follower and leader in their sector: $q_{fjt} = \left(\overline{q}_{jt}\right)^{\alpha} \left(\underline{q}_{jt}\right)^{1-\alpha}$.

There is a prospective entrant in each sector each period. The prospective entrant conducts R&D, ι^e_{jt}, to innovate on top of the follower's technology. Their prospective technology is such that

$$q^e_{jt+1} = D^e_{jt}\left(1 - \Phi^e\right)\lambda \underline{q}_{jt} + D^e_{jt}\Phi^e \overline{q}_{jt} + \left(1 - D^e_{jt}\right)\underline{q}_{jt},$$

where $D^e_{jt} = 1$ with probability ι^e_{jt} and $\Phi^e = 1$ with probability ϕ. They enter next period if their innovation is successful, and that their technology is higher than the follower's. They then replace the replace the follower in that sector. I assume that the entrant inherits the follower's customer capital stock.

The timing within a period is as follows. The duopolists first simultaneously set quantities, after which fringe firms set their quantities. Incumbents and entrants then proceed to set their innovation rates. Afterwards, firms realize their profits and pay wages to their workers. Finally, the outcomes of innovation and entry are realized.

3.2 Characterization and equilibrium

3.2.1 Households

In equilibrium, it can be shown that young households are on their Euler equation for firm bundles, while old households are not. Firms are then wholly owned by young households, hence discount future payoffs at the discount factor of young households, which is β. One result of this is that the firm discount rate is unaffected by movements in ϵ^O, a parameter I later vary to model aging demographics. This isolates the effect of aging demographics that operates through customer capital, as opposed to effects that operate through the firm discount rate (e.g. changes in the interest rate).

Demand for duopolist good i in sector j at time t for young and old households, given prices and habit stocks, can be derived as

$$C^Y_{ijt} = \frac{p^{-\rho}_{ijt}}{p^{1-\rho}_{ijt} + p^{1-\rho}_{-ijt} + \int^{\mathcal{N}} p_{fjt}\left(x\right)^{1-\rho} dx} \tag{6}$$

$$C^O_{ijt} = \frac{(k_{ijt})^\theta p_{ijt}^{-\rho}}{(k_{ijt})^\theta p_{-ijt}^{1-\rho} + (k_{-ijt})^\theta p_{-ijt}^{1-\rho} + (0.5)^\theta \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx}. \tag{7}$$

Moreover, the sectoral expenditure is $p_{jt}C_{jt} = 1$. Note that demand for good i in sector j only depends on prices of goods in that sector, and not on the aggregate price. Effectively, firms only compete with others in their own sector, disregarding firms outside their sector. This is a result of the assumption of linear labor disutility, combined with the outer nest elasticity of 1.

3.2.2 Firms

Fringe firms take their elasticity of demand as given. They charge a fix markup over their marginal cost, setting price as $p_{fjt}(x) = \frac{1}{q_{fjt}} \frac{\rho}{\rho-1}$.

Duopolists can affect their demand and demand elasticity, through the quantity that they produce and their customer capital. Define the sectoral market share of firm i in sector j at time t by $s_{ijt} \equiv p_{ijt}C_{ijt}/(p_{jt}C_{jt})$. Since sectoral expenditure $p_{jt}C_{jt} = 1$, this simplifies to $s_{ijt} = p_{ijt}C_{ijt}$. Given firms choices of quantities $(q_{ijt}l_{ijt}, q_{-ijt}l_{-ijt})$, market shares (s_{ijt}, s_{-ijt}) can be solved implicitly from

$$\begin{aligned} s_{ijt} = & \frac{1}{1 + \left(\frac{l_{ijt}}{s_{ijt}} \frac{s_{-ijt}}{l_{-ijt}} \frac{q_{ijt}}{q_{-ijt}}\right)^{1-\rho} + N \left(\frac{q_{ijt}}{q_{fjt}} \frac{\rho}{\rho-1} \frac{s_{ijt}}{l_{ijt}}\right)^{1-\rho}} M^Y \\ & + \frac{(2k_{ijt})^\theta}{(2k_{ijt})^\theta + (2k_{-ijt})^\theta \left(\frac{l_{ijt}}{s_{ijt}} \frac{s_{-ijt}}{l_{-ijt}} \frac{q_{ijt}}{q_{-ijt}}\right)^{1-\rho} + N \left(\frac{q_{ijt}}{q_{fjt}} \frac{\rho}{\rho-1} \frac{s_{ijt}}{l_{ijt}}\right)^{1-\rho}} M^O. \end{aligned} \tag{8}$$

Note that only relative productivities matter, so that market shares are functions of input choices, customer capital stocks, and productivity gap between the duopolists.

The problem of a duopolist can be written recursively as

$$\begin{aligned} v_t(k, k_-, m) = & \max_{l,\iota} s_t(l, l_-, k, k_-, m) - l - \frac{\gamma}{2} \left(\log\left(\frac{1}{1-\iota}\right)\right)^2 \\ & + \beta E_{m',\mathcal{R}} \left[v_{t+1}(k', k'_-, m')(1 - \mathcal{R})\right] \end{aligned} \tag{9}$$

with k', k'_- evolving according to equation (5). $\mathcal{R}$ is an indicator for if the firm is replaced by the entrant.

3.2.3 Equilibrium

I consider the recursive equilibrium on the balanced growth path (BGP) of the economy, where household mass M^Y, M^O are constant, aggregate consumption C^Y, C^O grow at a constant rate, and the distribution of sectors is stationary. Formally, the recursive equilibrium on the BGP consists of household policies $C^Y(k,k_-,m), C^O(k,k_-,m)$, A^Y, A^O, L^Y, L^O, firm policies $l(k,k_-,m), \iota(k,k_-,m)$, $\iota^e(k,k_-,m)$, firm value $v(k,k_-,m)$, distribution of sectors $\Omega(k,k_-,m)$, law of motion Γ for Ω, and relative prices P^A, $\frac{p_-}{p}(k,k_-,m)$, $\frac{p_f}{p}(k,k_-,m)$, such that

1. $C^Y(k,k_-,m), C^O(k,k_-,m), A^Y, A^O, L^Y, L^O$ solves the household problem, given prices
2. Given competitor's policies $l(k,k_-,m), \iota(k,k_-,m), \iota^e(k,k_-,m)$, the firm value $v(k,k_-,m)$ is consistent with the firm Bellman equation (9), and firm policies $l(k,k_-,m), \iota(k,k_-,m)$, $\iota^e(k,k_-,m)$ are consistent with maximization
3. P^A clears the asset market
4. Relative prices $\frac{p_-}{p}(k,k_-,m), \frac{p_f}{p}(k,k_-,m)$ clears the goods market for each sector
5. The distribution of sectors $\Omega(k,k_-,m)$ is stationary, and its law of motion, Γ, is consistent with firm policies: For all sets S in the Borel algebra of the domain of Ω, and for all states (k,k_-,m) with $k+k_-=1$,

$$\Omega(S) = \int \Big\{ 1_{\{(k'(k,k_-,m),1-k'(k,k_-,m),m\prime)\in S\}} \\ \times Pr\left(m'|\iota(k,k_-,m), \iota(k_-,k,-m), \iota^e(k,k_-,m), m\right) \Big\} d\Omega(k,k_-,m)$$

The equilibrium concept is standard, with the addition that duopolist behavior within a sector is strategic and constitutes a Markov Perfect Equilibrium.

3.3 Parameterization

The quantitative model has 11 parameters, $\left(\beta, \epsilon^Y, \epsilon^O, \rho, \delta, \theta, \lambda, \gamma, \phi, \mathcal{N}, \alpha\right)$. I describe how the parameters are set. Table 1 summarizes the parameterization.

I parameterize the model to correspond to targets for the US in the late 1970s. One period in the model corresponds to a quarter. I set the discount rate β at 0.99. ϵ^Y is set at 0.0357, which implies that the average age of young households is 27, assuming households enter the economy at 20 years old. ϵ^O is then set so that the population share of old households is 0.65, which is the average for the late 1970s.

I set ρ at 10. Coupled with the Cournot competition structure, the trade literature has shown that this generates empirically relevant cost pass through to prices (Amiti et al. 2019)[3].

[3] The cost pass through has a close relationship with how demand elasticity vary, which is important for the effect of customer capital as shown in the simple model

I set $\mathcal{N}$, the mass of fringe firms, so that average market share of a duopolist firm in the model is 0.26. α, the weight on the leading firm's productivity in fringe productivity, is set to target an aggregate markup of 1.28.

γ, the cost of R&D, affects the success rate of R&D. In the model, entry and exit of duopolist firms only occur after successful R&D by potential entrants. I set γ to target an exit rate of 1.82%, which corresponds to the exit rate of firms with 20 or more employees. λ, the innovation step size, can then be set to target a growth rate of 2.2%. I set ϕ, the probability of the less productive firm closing the gap after a successful innovation, to achieve 0.20 revenue productivity dispersion.

Param	Description	Value	Param	Description	Value
β	Discount rate	0.99	λ	Growth step size	1.065
ϵ^Y	Prob. of turning old	0.0357	$\mathcal{N}$	Mass of fringe	6.5
ϵ^O	Prob. of death	0.0192	α	Fringe productivity weight	0.808
ρ	Sectoral elas. of substitution	10	γ	Cost of R&D	4.05
δ	Depreciation of consumer habit	0.0133	ϕ	Prob of closing gap, upon success	0.212
			θ	Strength of consumer habit	2.2

Table 1: Parameterization

The parameters governing the strength and depreciation customer capital, θ and δ, are central to the model. I discipline them using estimates from Bronnenberg et al. (2012). I describe the identification argument there, and how it maps to my model.

Bronnenberg et al. (2012) leverage migration as a source of change in market conditions facing the individuals migrating, and track changes in migrating individuals' consumption patterns to inform the strength and depreciation of consumer habit. Consider two markets in different locations, A and B, with goods x and y as the two products with the highest market share in both markets. Define the relative share of good x in market A or B as the ratio of x's sales to the sum of x and y's sales in that market. Denote the relative share by S_A^x and S_B^x in market A and B respectively. The relative shares could differ across markets[4], but a key assumption is that individuals in either markets are not systematically different. Now take an individual i that migrates from A to B. Denote their relative expenditure on x, the ratio of their spending on x to the sum of their spending on x and y, at period t after migration by S_{it}^x. Before they migrate, their expenditure share on x would on average look the same as others in market A, i.e. $S_{i0}^x = S_A^x$. After they migrate, over a period of time, their expenditure share would on average look the same others as in market B, i.e. $\lim_{t\to\infty} S_{it}^x = S_B^x$. In the period right after they migrate, the extent to which S_{i1}^x is similar

[4]This could be from, for example, relative price differences due to supply costs or regional contracts, or differences in accessibility of product.

to S_A^x as opposed to S_B^x will inform the strength of consumption habits. Define $G_t \equiv \frac{S_A^x - S_{it}^x}{S_A^x - S_B^x}$. If there was no consumption habit, we would expect S_{i1}^x to immediately be the same as long-time residents in B, so $G_1 = 1$. Whereas if consumption habits were perfectly rigid, we would have $G_1 = 0$. Moreover, the time it takes after migration for G_t to rise to 1 will inform the depreciation of consumption habits.

The migration setup is analogous to where market conditions changes within a single market. Define a market state as a combination of good characteristics (relative prices and qualities) and customer capital. A long run market state is a market state where, if good characteristics are held constant, customer capital does not change over time. Markets in different locations could then be considered as different long run market states. Take A and B as different long run market states. S_A^x and S_B^x would correspond to long run relative market shares. Starting from A, suppose goods prices or qualities change such that the new long run state is B. Let S_{it}^x be the relative market share of x at period t after the change. The measure $G_t \equiv \frac{S_A^x - S_{it}^x}{S_A^x - S_B^x}$ could then be applied to discipline the strength and depreciation of habits.

I implement this in the model as follows. A long run market state in the model is a combination of relative prices and customer capital, $\left(\frac{p_-}{p}, \frac{p_f}{p}, k, k_-\right)$, such that holding constant relative prices, next period customer capital is the same as today's. I start with a sector where duopolists have the same productivity, so $m = 0$. Relative prices are set corresponding to equilibrium firm policies, i.e. $\left[\frac{p_-}{p}\left(k, k_-, m=0\right), \frac{p_f}{p}\left(k, k_-, m=0\right)\right]$, such that $k > 0.5$ and next period customer capital, given $\left(\frac{p_-}{p}, \frac{p_f}{p}, k, k_-\right)$, is unchanged. This corresponds to long run market state A in the above paragraph. I then change m to 1, and set prices according to equilibrium policies, i.e. $\left[\frac{p_-}{p}\left(k, k_-, m=1\right), \frac{p_f}{p}\left(k, k_-, m=1\right)\right]$. Holding these prices constant, I calculate $\left(\hat{k}, \hat{k}_-\right)$ that corresponds to the new long run market state - B in the above paragraph. I can then track S_{it}^x and calculate the measure G_t as above. I target a G_1 of 0.68, so that 68% of the difference in long term expenditure shares is reached upon price change. The remaining 32% is closed in subsequent years, and I target a half-life of 9.62 years. This implies a value of δ at 0.0133.

Table 2 shows model moments under the parameterization, compared to their targets. Construction of these moments are left to the appendix.

4 Empirical support for the model

Before going to the quantitative exercises, I provide empirical support for the model to show that the model results are sensible. I proxy for the strength of customer capital effects in an

Moment	Model	Target	Source
Revenue productivity dispersion	0.203	0.20	Compustat
Fraction of long term market share obtained upon price change	0.677	0.68	Bronnenberg et. al. (2012)
Aggregate markups	1.281	1.28	Compustat
Growth rate	2.22%	2.2%	SF Fed
Mean market share	0.265	0.26	Mongey (2021)
Entry rate	1.87%	1.82%	BDS

Table 2: Model moments

industry using the consumption share of older households within that industry. Using variations within industries over time, I find evidence that when the consumption share of older households is higher in an industry, there is larger divergence in R&D investment between the most productive firms and the rest for that industry. This supports the predictions from the model. I then run similar regressions on simulated data from the quantitative model, finding results of the same magnitude as the empirical regressions.

4.1 Effect of older households consumption share

The model predicts that changes to the firm's customer capital affects its spending on R&D. In the data, customer capital at the firm level is difficult to quantify. Instead, I take a step back and consider the strength of customer capital effects at the industry level, proxying for it using the consumption share of older households within an industry.

Bornstein 2021 documents that households above 35 are significantly less likely to switch products than those younger. Using retail scanner data, under the assumption that product quality and price for the same product are constant across markets in a given period, the author exploits variations in consumption of individual products across markets to identify the persistence of product consumption. This is done for various consumer age groups. Bornstein finds that households below 35 have significantly lower persistence compared to those above 35. This is true for a wide variety of product types.

The persistence in consumption can be interpreted as the strength of customer capital effects. Without customer capital effects, there would be no persistence once quality and price are fully accounted for. Persistence close to 1 implies strong customer capital effects, whereby households keep to their consumption habits despite movements in product quality and price. With persistence being larger for older households, when the consumption share of older households is larger, we would expect stronger customer capital effects on average for the industry. This motivates using the consumption share of older households as a proxy for the strength of customer capital effects.

The simplified model in section 2 predicts that, for revenue productivity dispersion that is not too large, stronger customer capital effects increases R&D spending for the most productive firms, and decreases R&D spending for other firms. This implies that the difference in R&D spending between the most productive firms and the rest increases, which I refer to as R&D divergence. Greater divergence leads to more dispersion in productivity as well. I now examine how the consumption share comoves with these measures in the data.

4.2 Data

I construct a panel data of industries from 1990 to 2019, with measures of the share of expenditures by older households, R&D spending, and revenue productivity dispersion for each industry. Measures for R&D spending and revenue productivity dispersion are constructed from public firms data (Compustat). The expenditure share is constructed from household data from the Consumer Expenditure Survey (CEX).

I first estimate revenue productivity for public firms. This allows me to separate firms within an industry into leaders and followers based on their revenue productivity. I estimate firm revenue productivity via production function estimation, following the method in Flynn et al. (2019). I specify firms' production function as a flexible translog in capital and inputs, and allow the coefficients to vary with time and 2 digit NAICS industries. Details are provided in the appendix.

An observation in my panel is a 3 digit NAICS industry at a given period. Industry revenue productivity dispersion is calculated as the standard deviation of revenue productivity of firms within the industry.

Regarding industry R&D spending, the model makes predictions on the level of innovation, which is a transformation of R&D spending. I consider 2 transformations. The first is using R&D spending directly, which corresponds to a linear cost of innovation. The second is using $\log(\text{R\&D} + 1)$, which allows for a convex cost of innovation. I consider firms above the 90^{th} quantile of revenue productivity in an industry as leaders, and firms below as followers. For the measure of innovation by the leader in an industry, I take the average of the innovation measure (R&D or $\log(\text{R\&D} + 1)$) across leaders in the industry. The measure of innovation by the follower is constructed similarly. R&D divergence is then the difference between the measure of innovation by the leader and the follower.

The industry older household expenditure share is constructed as the ratio of total value that households above 35 spend on the industry, to total value that all households spend on the industry. The CEX reports household consumption by modules, along with household characteristics. I crosswalk CEX modules to NAICS industries, largely following the BLS concordance. Details are given in the appendix. Following Bornstein (2021), I choose 35 as the age cutoff for older households[5].

I additionally filter and transform the data suit aspects of customer capital and innovation. The customer capital mechanism mainly affects consumer goods. To focus on such industries, I restrict my panel to industries that produce a large fraction of output as final goods, defined by being above the median industry in the economy in fraction of output that are final goods. Moreover, both customer capital and innovation are slow processes, hence I would want take out short run fluctuations in revenue productivity, R&D spending, and expenditure composition. I divide the sample period into bins of 3 years and take average values across the 3 years for each bin[6].

4.3 Comovements with consumption share of older households

Consider first the relationship between the consumption from older households and innovation. Equation (2) describes innovation as a function of customer capital and relative productivity. As such, I run the following regression

$$Y_{jt} = \beta_0 + \beta_1 S_{jt} + \beta_2 A_{jt} + \alpha_j + \eta_t + \epsilon_{jt}, \tag{10}$$

where j denotes industry and t denotes the 3-year period. S_{jt} is the share of expenditures by older households and A_{jt} is revenue productivity dispersion. Y_{jt} are measures of innovation, R&D or $\log(\text{R\&D} + 1)$, for the average leader, the average follower, and R&D divergence. Each measure of Y_{jt} is standardized. I include industry and time period fixed effects. β_1 is the main parameter of interest, and results are given in Table 3.

The consumption share of older households comoves positively with innovation by leaders. Regarding innovation by followers, the consumption share is largely uncorrelated, though the point estimates are small and negative. The consumption share also positively comoves with R&D divergence. For interpretation, the standard deviation of the consumption share, after controlling for industry and time period fixed effects, is 0.014. So a 1 standard deviation increase in the consumption share in an industry would, for example, be associated with an increase the average leader's $\log(\text{R\&D} + 1)$ in that industry by 0.14 standard deviation. The average increase in the consumption share of older households (due to aging demographics) from 1990 to 2019 is 0.06, which would be associated with an increase the average leader's $\log(\text{R\&D} + 1)$ by 0.61 standard deviation.

Now consider the relationship between the consumption from older households and productivity dispersion. As customer capital affects the innovation difference in leaders and

[5] similar for 50 as cutoff

[6] Results are similar when using 5 year bins instead

followers, which in turn affects the evolution of productivity dispersion, I run the following regression

$$\Delta Disp_{jt+1} = \gamma_0 + \gamma_1 S_{jt} + \psi_j + \zeta_t + \varepsilon_{jt}, \tag{11}$$

where $\Delta Disp_{jt+1}$ is the change in revenue productivity dispersion between 3-year period $t+1$ and 3-year period t. Results are in the last column of Table 3. The consumption share of older households comoves positively with changes in revenue productivity dispersion as well.

Dep var	Top 90^{th}		Bottom 90^{th}		Difference		$\Delta Disp_{jt+1}$
	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$	
S_{jt}	7.31 (1.84)	10.21 (3.08)	-0.33 (-0.16)	-1.46 (-0.64)	8.47 (1.80)	13.65 (2.52)	0.75 (2.65)
N ind	28	28	28	28	28	28	28
N ind×time	232	232	265	265	224	224	258

Note: T-stat in parentheses. Heteroskedastic robust standard errors.

Table 3: Consumption from older households and innovation

The direction of comovements in various measures with the consumption share from older households can be compared to the predictions from the model. The first matter is whether revenue productivity dispersion in the data is low enough that we expect the inequality in proposition 2 to hold. For values of the substitution parameter ρ often used, the log of the right hand side of the inequality in proposition 2 is around 0.66. For public firms, the standard deviation of log revenue productivity across firms, after taking out a 4 digit NAICS and time fixed effect for each firm, is 0.28[7]. This level of dispersion is low enough compared to thresholds associated with relevant values of the substitution parameter.

The model then predicts that higher consumption share from older households leads to more leader innovation, less follower innovation, more innovation divergence, and more revenue productivity dispersion, which the regression results support. More consumption from older households raises demand and lowers demand elasticity for the leader's goods, as a larger pool of older households with strong habit effects grow attach to the leader's goods. For the follower, their demand decreases as the pool of young, unattached households shrink. The changes in demand, as result of changes in customer capital, increases leader innovation and decreases follower innovation.

[7]The average of the dispersion measure for industries in my industry panel is similar, at 0.27

4.4 Comparison to the quantitative model

For empirical support for the quantitative results of the model, I run similar regressions to equations (10, 11) on model simulated data. I require variation in the share of older households in the model. Since the share is constant along the BGP, I rely on simulation along the transition of the economy as the share of older households change over time. Note, however, that this is aggregate variations in the consumption share, whereas the data features both aggregate variations and industry specific variations.

For the transition, I assume that the economy is on a BGP in 1960. I feed in a path of ϵ_o, the probability that old households are replaced, that generates a path of population of older households M_o that is the same as projected for the US until 2060, after which I assume ϵ_o remains constant. I then simulate sectors from 1960 to 2060, using equilibrium policies. Similar to the empirical data, I construct a panel of simulated sectors from 1990 to 2019, divide the sample period in the simulated panel into bins of 3 years and take average values across the 3 years for each bin.

I then run regressions in equations (10, 11) on the simulated panel, but without time fixed effects. Measures of R&D are also standardized for the simulated panel. Results are in table 4, along with counterparts from the empirical data. I include results from regressions both with and without time fixed effects for the empirical data, for better comparison.

The coefficients from the simulated panel are smaller than point estimates from the data, but similar in magnitude.

	Simulated	Empirical	
R&D	6.60	8.54	8.47
		$(2.34, 14.73)$	$(-0.84, 17.78)$
$\log(1 + \text{R\&D})$	6.62	8.84	13.65
		$(1.90, 15.78)$	$(2.94, 24.37)$
ΔDisp	0.28	0.48	0.75
		$(0.12, 0.85)$	$(0.19, 1.31)$
FE	Ind	Ind	Ind, Time

Note: 95% Confidence interval in parentheses. Heteroskedastic robust standard errors.

Table 4: Regression coefficients, simulated and empirical

5 The effect of aging demographics

I model aging demographics as a decrease in ϵ^O, the probability that old households are replaced, which leads to a rise in the population share of old households. I consider the effects of changes in aggregate customer capital arising from aging demographics on measures of R&D divergence, markups, concentration, entry/exit of firms, and growth. I show these effects through comparing the economy on different BGPs corresponding to different ϵ^O, and through the transition of the economy from one BGP to another.

5.1 Comparing BGPs

I compare the economy on the baseline BGP to one with a lower probability of death, $\epsilon^O = 0.0139$. This corresponds to the share of old households in the economy at 0.72, the average for the US in the 2010s. Results are in Table 5, with column 2 showing the changes from the baseline for the new BGP, and column 3 showing the empirical changes for comparison.

	Model		Data
Fraction of older households	0.65	0.72	
R&D divergence	0.0171	+0.115 std	+0.524 std
Revenue productivity dispersion	0.203	+0.053	+0.113
Aggregate markups	1.281	+0.074	+0.11
Mean market share	0.265	+0.032	+0.05
Entry/Exit rate	1.87%	-0.47%	-0.51%
Growth rate	2.22%	+0.04%	-0.36%

Table 5: Comparing Balanced Growth Paths

In the model, aging demographics increases the difference in innovation between leaders and followers, leading to an increase in revenue productivity dispersion, from 0.203 to 0.256. Leaders widen their productivity gap, allowing them to charge higher markups while gaining more market share from both the follower and the fringe. As a result, aggregate markups rises from 1.281 to 1.355, and the average market share for a duopolist firm rises from 0.265 to 0.297. The value of having either lower productivity, i.e. being a follower, or lower customer capital falls. Hence the entry rate declines, from 1.87% a year to 1.4%, as potential entrants are dis-incentivized to conduct R&D. The growth rate sees a small increase, from 2.22% annually to 2.24%.

The changes in these measures in the model, when compared across BGPs, are sizable with regards to actual changes observed in the data. The increase in R&D divergence and revenue productivity dispersion in the model is around 20% and 45% observed in the data. For aggregate markups and mean market share, they are both around 65% of the increased observed empirically. The decline in exit rate generated is around 90% of the empirical decline. The model misses the growth rate though, generating a small increase in growth while actual growth has declined.

The changes in these measures across BGPs stem from aging demographics altering customer capital and firm demand, hence affecting firm innovation incentives. Aging demographics induces a higher consumption share from older households at the aggregate level, for all sectors. As discussed in subsection 4.3, the higher share raises demand and lowers demand elasticity for the leader, incentivizing the leader to innovate. While for the follower, it lowers demand, dis-incentivizing the follower to innovate.

5.2 On the transition

Comparing BPGs alone does not give the full picture. Aging demographics is a slow process, and the movement to a new BGP may be slow as well. Moreover, the degree of demographic aging extends beyond level in the 2010s, and forward looking firms take this into account. Here, I look at the transition from one BGP to another. I assume that the economy is initially on the BGP in 1960, with the parameters in subsection 3.3. I then feed in the path of ϵ^O that generates the projected path of the population share of older households to 2060. I assume that ϵ^O is constant after 2060.

Figure 1 shows the result of this transition, for the years 1980 to 2060. Panel (a) shows the realized and projected path of the population share of older households in the economy. Panel (b) shows the evolution of the entry/exit rate. It decreases over time, with the decrease slowing down once the share of older households stabilize. Panel (c) shows the evolution of the labor productivity growth rate. It fluctuates, and eventually increases to a higher level.. Panel (d) shows the evolution of aggregate markups. It increases slowly, with the increase continuing even after the share of older households stabilize.

Table 6 shows the changes from 1980 to 2020 implied by the transition. While qualitatively similar to the BGP comparison (table 5), the changes along the transition for revenue productivity dispersion, aggregate markups, and mean market share are quantitatively smaller. These measures are largely dependent on the distribution of sectors. The distribution is slow moving, as the customer capital stock is persistent and slow to increase, and the productivity gap is slow to change. Of these three measures the increase in mean market share is the closets to its BGP comparison counterpart, whereby leaders expand their sales along the transition in order to capture aging consumers. Meanwhile R&D divergence and the entry/exit rate see larger changes than their BGP comparison counterparts. These two measures are more closely tied to firm value, which is fast to change since it is forward looking and tracks future changes in the share of older households.

Compared to changes observed in the data, the increase in revenue productivity disper-

Figure 1: Transition

sion, aggregate markups, and mean market share along the transition is around 10%, 20% and 35% respectively. The increase in R&D divergence is around 30% observed in the data, and the decline in exit rate is around 100% of the empirical decline.

Year	Model		Data
	1980	2020	1980-2020 change
R&D divergence	0.0178	+0.151 std	+0.524 std
Revenue productivity dispersion	0.203	+0.01	+0.113
Aggregate markups	1.28	+0.02	+0.11
Mean market share	0.264	+0.017	+0.05
Entry/Exit rate	1.86%	-0.51%	-0.51%

Table 6: Transition

6 Response of firms to government policies

The section above suggests a quantitative importance of interaction between customer capital and innovation. In this section, I consider how firm response to government policies can differ once customer capital is accounted for. I first discuss the inefficiencies associated with the model environment, to highlight how the government could pursue policies to improve upon the equilibrium. I then consider 2 innovation policies: a subsidy to entry, and subsidy to R&D. The 2 policies are widely used, as different means of raising innovation rate. For each policy, I also compare its effect to an environment without customer capital.

6.1 Welfare and market inefficiencies

Define social welfare as the integral over the utility of all of households, both alive and yet to be born. I assume that the discount rate for future generations is the same as the household time discount rate, β. Social welfare at time t can then be written as

$$SW_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(M_s^Y U_s^Y + M_s^O U_s^O \right) \right],$$

with $M_{y,s}, M_{o,s}$ being the mass of young and old households at time s, and U_s^Y, U_s^O are the utilities of young and old households at time s. For a planner that maximizes social welfare, they choose the amount supplied of each good, along with innovation rates for each duopolist firm and potential entrant. They face the same allocation of goods among young and old household for each amount supplied as in the market equilibrium, along with the same law of motion for habits as in the market equilibrium. I leave the specific formulation for these restrictions to the appendix.

The planning solution differ from the market equilibrium in both amount of good supplied and innovation rates. Consider the planning problem and the market equilibrium on the balanced growth path. The linear disutility of labor combined with log utility on the nested CES consumption allows the planner to maximize utility for each sector individually. The problem for a specific sector can be written recursively as

$$\begin{aligned} W(m,k) = & \max_{l^{lead}, l^{follow}, l^{fringe}, \iota^{lead}, \iota^{follow}, \iota^{ent}} S\left(l^{lead}, l^{follow}, l^{fringe}, m, k\right) - \left(l^{lead} + l^{follow} + \mathcal{N} l^{fringe}\right) \\ & - \frac{\gamma}{2}\left(\left(\log\left(\frac{1}{1-\iota^{lead}}\right)\right)^2 + \left(\log\left(\frac{1}{1-\iota^{follow}}\right)\right)^2 + \left(\log\left(\frac{1}{1-\iota^{ent}}\right)\right)^2\right) \\ & + \beta E\left[W(m',k') + \frac{1}{1-\beta}\ln(\lambda)\mathcal{I}\right], \end{aligned} \tag{12}$$

subject to the law of motion for customer capital k. The planner chooses the amount of labor for the leader, follower, and fringe, $\left(l^{lead}, l^{follow}, l^{fringe}\right)$, which maps to the amount of goods produced when given the productivity level. The planner also chooses the innovation rates for the leader, follower, and potential entrant, $\left(\iota^{lead}, \iota^{follow}, \iota^{ent}\right)$. Here m is the productivity steps between the leader and follower, and k is the level of customer capital for the leader. $S\left(l^{lead}, l^{follow}, l^{fringe}, m, k\right)$ is the utility of households from consumption,

$$S(\mathcal{X}) = M_y\left[\ln\left(\left[C^{Y,lead}(\mathcal{X})^{\frac{\rho-1}{\rho}} + C^{Y,follow}(\mathcal{X})^{\frac{\rho-1}{\rho}} + \mathcal{N}C^{Y,fringe}(\mathcal{X})^{\frac{\rho-1}{\rho}}\right]\right)^{\frac{\rho}{\rho-1}}\right]$$
$$+ M_o\left[\ln\left(\left[(2k)^{\frac{\theta}{\rho}} C^{O,lead}(\mathcal{X})^{\frac{\rho-1}{\rho}} + (2-2k)^{\frac{\theta}{\rho}} C^{O,follow}(\mathcal{X})^{\frac{\rho-1}{\rho}} + \mathcal{N}C^{O,fringe}(\mathcal{X})^{\frac{\rho-1}{\rho}}\right]\right)^{\frac{\rho}{\rho-1}}\right],$$

where $\mathcal{X} = \left(l^{lead}, l^{follow}, l^{fringe}, m, k\right)$, and consumption allocations $C^{Y,lead}$, $C^{Y,follow}$, $C^{Y,fringe}$, $C^{O,lead}$, $C^{O,follow}$, $C^{O,fringe}$ are constrained to be the same as in the market equilibrium, given $\mathcal{X}$. $W(m,k)$ is the maximal utility for the sector, less a technology term $\frac{1}{1-\beta}\ln q^{lead}$[8].

We can compare the planner's choice of production and innovation to the market equilibrium. First consider the case without customer capital. The choice of production is static problem. Firms supply less compared to the planner's choice and charge positive markups. For innovation rates, the payoff of innovating for the firm differs from the planner in two ways. One, the per-period gains differ, as the planner considers the consumption gains from higher productivity and production, while the firm only reaps the gains to profits. Two, while the planner enjoys the gains from higher productivity forever, the firm only benefits from a higher productivity gap until its rival or potential entrant innovates.

With customer capital, firms have an incentive to increase supply to build customer capital. This incentive weakens as firms widen the productivity gap with its rival. Customer capital affects the gains in profit, hence firm innovation changes as well. Customer capital also changes consumption utility, inducing the planner to focus production as well as innovation more on the leader. Whether there is under supply and/or under-innovation by firms is not theoretically definitive, however numerical results under baseline parameters point to large under supply and under-innovation.

This leaves room for the government to improve upon the market allocations. For example, the government can utilize a mixture of targeted production subsidy to induce higher production, along with targeted R&D subsidy to induce higher innovation.

6.2 Entry subsidy and R&D subsidy

I consider the effects of a subsidy to entry and a subsidy to R&D cost. The subsidies are meant to induce higher innovation, with the former for entrant innovation and the latter for incumbent innovation. These subsidies are untargeted, in that firms with different productivity gaps and different levels of customer capital receive same proportional subsidy. I enact a 10% subsidy to potential entrants' cost of innovation, and 10% subsidy to incumbent cost

[8] Maximal utility for the sector depends on the level of productivity, and equals $W(m,k) + \frac{1}{1-\beta}\ln q^{lead}$

of innovation respectively. Each policy is funded by lump-sum taxes on households. For the results, welfare is in consumption equivalence, and is calculated from the period when policy is announced and enacted, along the transition[9]. Other statistics are calculated on the BGP.

Consider first the entry subsidy. The first two columns of table 7 gives the percentage deviation from the baseline for various statistics, for the environment with and without customer capital respectively. The subsidy induces a higher entry rate and growth rate, along with lower concentration and markups. In unequal sectors, successful entrants innovate upon the follower technology, closing the productivity gap with the leader, leading these sectors to be less unequal. In neck-and-neck sectors, where incumbents have the same level of productivity, successful entrants become the leader, creating a productivity gap in these sectors. For our parameters, the majority of sectors are equal, and the first effect dominates.

Compared to its effect in an environment without customer capital, the entry subsidy has larger impact on concentration and markups. Two factors lead to this. First, in neck-and-neck sectors, the successful entrant, though with higher productivity, have lower customer capital than its rival. The low customer capital leader innovates less, due to the lower gains in profits when customer capital is low, and charge lower markups in order to build up its capital stock. For our parameters, these leaders lower their markups so much that there revenue productivity fall below their rivals', so much so that the average revenue productivity dispersion between firms within a sector increase.

The second factor is from a feedback effect, alluded to in subsection 2.2. In the environment with customer capital, the decreased productivity gap leads to lower customer capital accumulated by the leaders. Leaders then do lower innovation then previously, which over time further decrease the productivity gap.

The welfare gains from the entry subsidy are similar with and without customer capital, under our parameters. In the environment with customer capital, while the increase in growth on the BGP is weaker, the initial increase in growth on the transition is stronger, persisting for over 160 periods. Moreover, lower aggregate markups implies higher consumption, conditional on the level of productivity.

Now consider the R&D subsidy. Its effect on various statistics are given in the last two columns of table 7. The R&D subsidy induces an increase in the growth rate, along with higher concentration and markups. Leaders and followers increase their innovation proportionally from the subsidy, leading to an increase in the absolute difference in their innovation. Over time, this increases the productivity gap between leaders and followers.

Like the entry subsidy, the R&D subsidy have a larger impact on concentration and markups in the environment with customer capital. This similarly stems from the feedback

[9]Steady state welfare cannot be compared, as the productivity level matters.

	10% subsidy to entry cost		10% subsidy to R&D cost	
	With customer capital	Without customer capital	With customer capital	Without customer capital
Revenue productivity dispersion	+0.71%	-1.09%	+9.38%	+1.66%
Mean market share	-1.04%	-0.45%	+2.84%	+1.11%
Aggregate Markups	-0.46%	-0.21%	+1.32%	+0.39%
Entry rate	+8.20%	+6.52%	-5.55%	+0.41%
Growth rate	+0.08%	+0.21%	+8.64%	+8.62%
Welfare (CE)	+0.13%	+0.12%	+4.24%	+4.15%

Table 7: Entry and R&D subsidy

effect. The higher productivity gap allows leaders to accumulate more customer capital, which incentivize them to increase innovation. This furthers widen the productivity gap over time.

The impact on welfare is large, and similar between the two environments. The large effect is a feature in this class of endogenous growth models. The R&D subsidy raises the incentives to innovate, which directly affect the BGP growth rate, translating to a large increase in welfare.

7 Conclusion

This paper studies how customer capital affects firm innovation decisions the resulting consequences on aggregate productivity and concentration. I develop a step-by-step model of innovation incorporated with consumption habits. Consumption habits are the basis for customer capital for firms: a firm invests in customer capital by increasing production and lowering price in the current period, in order to enjoy higher and more inelastic demand in future periods. Through changing future demand, customer capital affects the firm's incentives to innovate. Changes in firm innovation then drives movements in industry concentration and aggregate markups.

I use the model to quantify the effects of changes in aggregate customer capital arising from aging demographics. By shifting demand composition towards old households who have strong habit effects, aging demographics induces the most productive firms in each industry to innovate more relative to their competitors. This results in rising concentration and markups. In the calibrated model, the induced rise in the share of older households in aggregate demand can account for 10% to 35% of the increase in divergence of R&D spending across firms, increase in revenue productivity dispersion across firms, and the rise in aggregate markups and industry concentration. The model suggests that these trends will continue, even after demographics have stabilize.

I also use the model to analyze how customer capital affects the outcomes of government innovation subsidies. Compared to an environment without customer capital, the effect of these innovation policies on productivity dispersion, concentration, and markups is around 2 to 3 times as large. This amplification arises from a feedback effect, whereby changes to a firm's productivity would, over time, affect its customer capital stock, which in turn alters its innovation choices.

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A Revenue productivity estimation

I estimate revenue productivity following Flynn et al. (2019). The approach uses a proxy estimator to estimate the production function (Ackerberg et al. (2015)), but with an additional restriction on returns to scale, which is necessary for identification. I assume a translog production function

$$y_{it} = \theta_t^v v_{it} + \theta_t^k k_{it} + \theta_t^{vv} v_{it}^2 + \theta_t^{kk} k_{it}^2 + \theta_t^{vk} v_{it} k_{it} + a_{it} + \epsilon_{it},$$

where y_{it} is log revenue, v_{it} is log cost of goods sold, k_{it} is log capital, and a_{it} is log revenue productivity. As in De Loecker et al. (2020), I allow for time-varying production function parameters, and estimate separately for each 2 digit NAICS sector.

k_{it} and v_{it} may be correlated with a_{it}, which gives rise to a simultaneity problem if we proceed to estimate the above function via OLS. The key insight is that a_{it} can be expressed as a function of the firm's observables, obtained from inverting out input demand:

$$a_{it} = \omega_t \left(v_{it}, k_{it}, z_{it} \right),$$

where z_{it} captures other factors that affect demand. Output can then be written as

$$y_{it} = \phi_{it} \left(v_{it}, k_{it}, z_{it} \right) + \epsilon_{it}.$$

For a given guess of $\theta_t = \left\{ \theta_t^v, \theta_t^k, \theta_t^{vv}, \theta_t^{kk}, \theta_t^{vk} \right\}$, one can obtain a guess of revenue productivity as

$$\tilde{a}_{it} \left(\theta_t \right) = \phi_{it} \left(v_{it}, k_{it}, z_{it} \right) - \left(\theta_t^v v_{it} + \theta_t^k k_{it} + \theta_t^{vv} v_{it}^2 + \theta_t^{kk} k_{it}^2 + \theta_t^{vk} v_{it} k_{it} \right).$$

I assume a Markov productivity process $a_{it} = g \left(a_{it-1}, \hat{\psi}_{it-1} \right) + \eta_{it}$, where $\hat{\psi}_{it-1}$ is the predicted probability that the firm continues to be in the sample. This gives one moment condition for θ_t:

$$\mathbb{E} \left[k_{it} \eta_{it} \right] = 0.$$

I impose the additional conditions that the return to scale is 1, which gives 3 more moments:

$$\begin{aligned} \mathbb{E} \left[v_{it} \left(RTS_{it} \left(\theta_t \right) - 1 \right) \right] &= 0 \\ \mathbb{E} \left[k_{it} \left(RTS_{it} \left(\theta_t \right) - 1 \right) \right] &= 0 \\ \mathbb{E} \left[\left(RTS_{it} \left(\theta_t \right) - 1 \right) \right] &= 0, \end{aligned}$$

where $RTS_{it} \left(\theta_t \right) = \theta_t^v + \theta_t^k + 2\theta_t^{vv} v_{it} + 2\theta_t^{kk} k_{it} + \theta_t^{vk} v_{it} k_{it}$.

B Proofs and derivations

Proposition 3. *For the 2 period industry duopoly,*

a) Second period payoff π_i is given by

$$\pi_i\left(k_{i2}/k_{-i2}, q_{i2}/q_{-i2}\right) = \frac{\left(\frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\left(\frac{q_{i2}}{q_{-i2}}\right)^{(\rho-1)/\rho} + \frac{1}{\rho}\right)\frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\left(\frac{q_{i2}}{q_{-i2}}\right)^{(\rho-1)/\rho}}{\left[1+\frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\left(\frac{q_{i2}}{q_{-i2}}\right)^{(\rho-1)/\rho}\right]^2};$$

b) For small $(\iota_{i1}, \iota_{-i1})$, first period R&D decision ι_{i1} is approximated by

$$\iota_{i1} = \frac{1}{\gamma}\left[\pi_2\left(k_{i2}/k_{-i2}, \lambda q_{i1}/q_{-i1}\right) - \pi_2\left(k_{i2}/k_{-i2}, q_{i1}/q_{-i1}\right)\right].$$

Proof. In the last period, given habit stocks, firm i solves

$$\max_{c_{i2}}\left[k_{i2}^{\theta/\rho}\left(\frac{c_{i2}}{c_2}\right)^{-1/\rho} c_2^{-1} - \frac{1}{q_{i2}}\right]c_{i2}$$

FOCs give

$$\frac{\rho-1}{\rho} - \frac{1}{q_{i2}}k_{i2}^{-\theta/\rho}\left(\frac{c_{i2}}{c_2}\right)^{1/\rho} c_2 - \frac{\rho-1}{\rho}k_{i2}^{\theta/\rho}\left(\frac{c_{i2}}{c_2}\right)^{\frac{\rho-1}{\rho}} = 0$$

Define inverse markup $\mu_{i2}^{-1} = \frac{1}{q_{i2}}k_{i2}^{-\theta/\rho}\left(\frac{c_{i2}}{c_2}\right)^{1/\rho}$ and revenue share $s_{i2} = k_{i2}^{\theta/\rho}\left(\frac{c_{i2}}{c_2}\right)^{\frac{\rho-1}{\rho}}$. We then have the system

$$1 - \frac{\rho}{\rho-1}\mu_{i2}^{-1} - s_{i2} = 0$$
$$1 - \frac{\rho}{\rho-1}\mu_{-i2}^{-1} - s_{-i2} = 0$$
$$s_{i2} + s_{-i2} = 1$$
$$\frac{\mu_{i2}}{\mu_{-i2}} = \frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\frac{q_{i2}}{q_{-i2}}\left(\frac{c_{i2}}{c_{-i2}}\right)^{-1/\rho}$$

This can be solved for to obtain, where $\frac{q_{i2}}{q_{-i2}} = \lambda^m$

$$\mu_i = \frac{\rho}{\rho-1}\left[1+\frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\lambda^{m(\rho-1)/\rho}\right]$$

so that

$$\pi_i = \left[1 + \frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\lambda^{m(\rho-1)/\rho}\right]^{-2}\left(\frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\lambda^{m(\rho-1)/\rho} + \frac{1}{\rho}\right)\left(\frac{k_{i2}^{\theta/\rho}}{k_{-i2}^{\theta/\rho}}\lambda^{m(\rho-1)/\rho}\right)$$

First period innovation approximation is obtained from FOCs and taking ι_{-i1} to 0. □

Proposition 4. $\frac{\partial^2 \pi_2(\kappa_i, m_i)}{\partial m_i \partial \kappa_i} > 0$ *iff*

$$\kappa_i^{1/\rho}\lambda^{m_i(\rho-1)/\rho} < \sqrt{4\left(1-\frac{1}{\rho}\right)^2\left(2-\frac{1}{\rho}\right)^{-2} + \frac{1}{\rho}} + 2\left(1-\frac{1}{\rho}\right)\left(2-\frac{1}{\rho}\right)^{-1}.$$

Proof. From the expression of π above, taking partial derivative wrt m yields

$$\frac{\partial \pi}{\partial m} = \left(\kappa_i^{1/\rho}\lambda^{m(\rho-1)/\rho}\left(\rho-1\right)/\rho \ln\lambda\right)\left[1+\kappa_i^{1/\rho}\lambda^{m(\rho-1)/\rho}\right]^{-3}\left[\kappa_i^{1/\rho}\lambda^{m(\rho-1)/\rho}\left(2-\frac{1}{\rho}\right)+\frac{1}{\rho}\right]$$

Further taking partial derivative wrt to $\kappa_i^{1/\rho}$ yields an expression with the same sign as

$$-\kappa_i^{2/\rho}\lambda^{2m(\rho-1)/\rho}\left(2-\frac{1}{\rho}\right) + \kappa_i^{1/\rho}\left(4\lambda^{m(\rho-1)/\rho}\left(1-\frac{1}{\rho}\right)\right) + \frac{1}{\rho}$$

Let $y = \kappa_i^{1/\rho}\lambda^{m(\rho-1)/\rho}$. The above reduces to

$$-\left(y - 2\left(1-\frac{1}{\rho}\right)\left(2-\frac{1}{\rho}\right)^{-1}\right)^2 + 4\left(1-\frac{1}{\rho}\right)^2\left(2-\frac{1}{\rho}\right)^{-2} + \frac{1}{\rho}$$

Apply the quadratic formula to solve for y, and setting it to be positive yields the condition. □

C Alternative discrete choice setup

C.1 Discrete choice in the simple model (section 2)

The results in section 2 remain if I instead have each household consuming only one of the two duopolist goods, with their choice of good being subjected to idiosyncratic preference shocks.

The utility of household h from consuming c_i amount of good $i \in \{1, 2\}$ is

$$u\left(\exp\left(\frac{\epsilon_i^h + \theta \log k_i}{\rho - 1}\right)c_i\right),$$

where $u(.)$ is a strictly increasing function, ϵ_i^h is the idiosyncratic preference shock of household h for good i, and k_i is the customer capital for good i, which is common to all households. The preference shock follows a Gumbel distribution, and is i.i.d. across products and across households. Once a household chooses to consume a good, they spend their whole endowment on that good, so $c_i = 1/p_i$. The household problem is then

$$U^h = \max_{i\in\{1,2\}} u\left(\exp\left(\frac{\epsilon_i^h + \theta \log k_i}{\rho - 1}\right)\frac{1}{p_i}\right).$$

The optimal choice i^h solves

$$i^h = \arg\max_i \exp\left(\frac{\epsilon_i^h + \theta \log k_i}{\rho - 1}\right)\frac{1}{p_i}$$

$$i^h = \arg\max_i (1-\rho)\log p_i + \theta \log k_i + \epsilon_i^h.$$

With the shocks ϵ_i^h drawn independently from the Gumbel distribution, the probability of good i being choosen is

$$\frac{k_i^\theta p_i^{1-\rho}}{k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}}.$$

Since the measure of households is 1, and each household who chooses good i spend 1 unit on the good, total expenditure on good i across all households is

$$p_i C_i = \frac{k_i^\theta p_i^{1-\rho}}{k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}},$$

where C_i is total quantity consumed of good i. Rearranging,

$$p_i = C_i^{\frac{-1}{\rho}} k_i^{\frac{\theta}{\rho}} \left[k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}\right]^{\frac{-1}{\rho}}.$$

With a similar expression for $-i$, we have

$$k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho} = k_i^\theta C_i^{\frac{\rho-1}{\rho}} k_i^{\frac{\theta(1-\rho)}{\rho}} \left[k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}\right]^{\frac{\rho-1}{\rho}} + k_{-i}^\theta C_{-i}^{\frac{\rho-1}{\rho}} k_{-i}^{\frac{\theta(1-\rho)}{\rho}} \left[k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}\right]^{\frac{\rho-1}{\rho}}$$

$$\left[k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}\right]^{\frac{1}{\rho}} = k_i^{\frac{\theta}{\rho}} C_i^{\frac{\rho-1}{\rho}} + k_{-i}^{\frac{\theta}{\rho}} C_{-i}^{\frac{\rho-1}{\rho}}.$$

Substituting back, we obtain the same inverse demand as in equation (1).

C.2 Discrete choice in the quantitative model (section 3)

I show that firms face the same demand and same optimization problem under an alternative environment where households in each period consume a single good in each sector, with their choice of goods subjected to preference shocks.

Household h aggregate goods according to

$$C_t^{ah} = \exp\left\{\int \ln\left[\exp\left(\frac{\epsilon_{ijt}^h + \theta \log k_{ijt}^a}{\rho - 1}\right) c_{ijt}\right] dj\right\},$$

where i denotes the good that the household consumes in sector j, with c_{ijt} being the quantity consumed. ϵ_{ijt}^h is the idiosyncratic preference shock of household h for good i in sector j in period t, and k_{ijt}^a is the customer capital for good i for household of type a, which is common to all households of that type. Good i that the household chooses in sector j can be a good produced by one of the two dominant firms in the sector, or a good produced by a fringe firm in the sector. The preference shock follows a Gumbel distribution, and is i.i.d. across goods and across households. Customer capital for goods for young households are constant at 0.5, $k_{ijt}^Y = 0.5\, \forall i, j$. Customer capital for goods produced by fringe firms are also constant at 0.5. Customer capital for either goods produced by dominant firms in the sector evolves according to equation (5).

Given their choice of good in each sector, the household allocate spending to each sector by solving

$$\max \int \ln\left[\exp\left(\frac{\epsilon_{ijt}^h + \theta \log k_{ijt}^a}{\rho - 1}\right) c_{ijt}\right] dj - L_t + \beta \mathbb{E} W_t^a\left(A_{t+1}^a\right),$$

$$\text{s.t.} \qquad \int p_{ijt} c_{ijt} dj + P_t^A A_{t+1}^a = L_t + \left(P_t^A + d_t\right) A_t^a.$$

First order conditions give that $p_{ijt} c_{ijt} = 1$, so that $c_{ijt} = 1/p_{ijt}$. Household h's choice of good for sector j then solves

$$\arg\max_i \,(1-\rho)\log p_{ijt} + \theta \log k_{ijt}^{ah} + \epsilon_{ijt}^h.$$

With the shocks ϵ_{ijt}^h drawn independently from the Gumbel distribution, the probability of good i being choosen, when i is produced by a dominant firm, is

$$\begin{cases} \frac{p_{ijt}^{1-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^N p_{fjt}(x)^{1-\rho} dx} & \text{if } h \text{ is Young} \\ \frac{(k_{ijt})^\theta p_{ijt}^{-\rho}}{(k_{ijt})^\theta p_{-ijt}^{1-\rho} + (k_{-ijt})^\theta p_{-ijt}^{1-\rho} + (0.5)^\theta \int^N p_{fjt}(x)^{1-\rho} dx} & \text{if } h \text{ is Old} \end{cases}.$$

Since the measure of households is 1, and each household who chooses good i in sector j spend 1 unit on the good, total expenditure on good i across households of type young and old is

$$p_{ijt}C^Y_{ijt} = \frac{p_{ijt}^{1-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}$$

$$p_{ijt}C^O_{ijt} = \frac{(k_{ijt})^{\theta}\, p_{ijt}^{-\rho}}{(k_{ijt})^{\theta}\, p_{-ijt}^{1-\rho} + (k_{-ijt})^{\theta}\, p_{-ijt}^{1-\rho} + (0.5)^{\theta} \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}.$$

Dividing by p_{ijt} gives the demand equations (6,7), with customer capital k_{ijt} evolving according to equation (5), so that the firm problem remains the same.

D Incorporating advertising

Advertising is a large component of firm spending that contributes to firm brand and help to boost demand for the firm. Recent works have explored how advertising affects innovation. In this section, I incorporate advertising into the quantitative model of section 3 and conduct the aging demographic exercise as in subsection 5.1.

D.1 Environment

The treatment of advertising follows closely from Cavenaile et al. (2024). Each period, dominant firms can spend on advertising in order to influence household demand for their good in that period.

Household aggregates goods within a sector j by

$$C^Y_{jt} = (\left(\frac{1+\omega_{1jt}}{1+0.5\left(\omega_{1jt}+\omega_{2jt}\right)}\right)^{\frac{1}{\rho}} \left(C^Y_{1jt}\right)^{\frac{\rho-1}{\rho}} + \left(\frac{1+\omega_{2jt}}{1+0.5\left(\omega_{1jt}+\omega_{2jt}\right)}\right)^{\frac{1}{\rho}} \left(C^Y_{2jt}\right)^{\frac{\rho-1}{\rho}} + \int^{\mathcal{N}} C^Y_{fjt}(x)^{\frac{\rho-1}{\rho}}\, dx)^{\frac{\rho}{\rho-1}}$$

$$C^O_{jt} = ((2k_{1jt})^{\frac{\theta}{\rho}} \left(\frac{1+\omega_{1jt}}{1+0.5\left(\omega_{1jt}+\omega_{2jt}\right)}\right)^{\frac{1}{\rho}} \left(C^O_{1jt}\right)^{\frac{\rho-1}{\rho}} + (2k_{2jt})^{\frac{\theta}{\rho}} \left(\frac{1+\omega_{2jt}}{1+0.5\left(\omega_{1jt}+\omega_{2jt}\right)}\right)^{\frac{1}{\rho}} \left(C^O_{2jt}\right)^{\frac{\rho-1}{\rho}} + \int^{\mathcal{N}} C^O_{fjt}(x)^{\frac{\rho-1}{\rho}}\, dx)^{\frac{\rho}{\rho-1}},$$

where $\omega_{1jt}, \omega_{2jt}$ are advertising efforts by firm 1 and 2. Household aggregation over sectors, their preference over aggregated consumption and labor, their budget constraint, and the evolution of habits remain the same as in section 3. Good demand is then

$$C^{Y}_{ijt} = \frac{\left(\frac{1+\omega_{ijt}}{1+0.5(\omega_{ijt}+\omega_{-ijt})}\right) p_{ijt}^{-\rho}}{\left(\frac{1+\omega_{ijt}}{1+0.5(\omega_{ijt}+\omega_{-ijt})}\right) p_{ijt}^{1-\rho} + \left(\frac{1+\omega_{-ijt}}{1+0.5(\omega_{ijt}+\omega_{-ijt})}\right) p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}$$

$$C^{O}_{ijt} = \frac{\left(\frac{1+\omega_{ijt}}{1+0.5(\omega_{ijt}+\omega_{-ijt})}\right) (2k_{ijt})^{\theta} p_{ijt}^{-\rho}}{\left(\frac{1+\omega_{ijt}}{1+0.5(\omega_{ijt}+\omega_{-ijt})}\right) (2k_{ijt})^{\theta} p_{-ijt}^{1-\rho} + \left(\frac{1+\omega_{ijt}}{1+0.5(\omega_{ijt}+\omega_{-ijt})}\right) (2k_{-ijt})^{\theta} p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}.$$

For dominant firm i in sector j in period t, they choose advertising effort for the period ω_{ijt} at cost $\frac{\chi}{\phi}\omega_{ijt}^{\phi}$. Advertising decisions are made simultanously with quantity supply decisions. Aside from advertising, everything else remain the same as in section 3.

D.2 Parameterization

The additional parameters χ, ϕ associated with advertising are calibrated to match an inverse U relationship between advertising expenditure and market share. In the data, for the 1970 to 1990 period, I run the regression

$$\log(1+\text{ad}_{ijt}) = \alpha_0 + \alpha_1 \text{share}_{ijt} + \alpha_2 \text{share}^2_{ijt} + \alpha_3 \log(1+\text{R\&D}_{ijt}) + \gamma_i + \delta_t + \epsilon_{ijt}.$$

The regression is at the firm level. i, j, t denotes firm, NAICS 4 digit sector, and year respectively. ad_{ijt} is spending on advertising, share_{ijt} is the ratio of firm i's sale in year t to the total sales in sector j in year t, and $\text{R\&D}_{ijt}$ is spending on R&D. I include firm and year fixed effects. I then simulate data from the model on the BGP, and run the same regression. I choose χ, ϕ so that the coefficients α_1, α_2 from the regression on simulated data matches that from the regression on empirical data.

Other parameters are set as described in subsection 3.3. Table 8 gives the full set of parameter values.

D.3 Aging demographics BGP comparison

Table 9 compares the BGP changes due to an increase in the share of old households, between the environments with and without advertisement. The changes are similar.

E Construction of moments

I detail the sources and computation for target moments from the data in subsection 3.3.

Param	Description	Value	Param	Description	Value
β	Discount rate	0.99	λ	Growth step size	1.065
ϵ_y	Prob. of turning old	0.0357	$\mathcal{N}$	Mass of fringe	6.5
ϵ_o	Prob. of death	0.0192	α	Fringe productivity weight	0.808
ρ	Sectoral elas. of substitution	10	γ	Cost of R&D	4.25
δ	Depreciation of consumer habit	0.0133	ϕ	Prob of closing gap, upon success	0.272
			θ	Strength of consumer habit	2.2
			χ	Shifter on advertising cost	0.0002
			ϕ	Curvature of advertising cost	2.7

Table 8: Parameters

	Without ad	With ad
R&D divergence	+0.115 std	+0.101 std
Revenue productivity dispersion	+0.053	+0.064
Aggregate markups	+0.074	+0.094
Mean market share	+0.032	+0.036
Entry/Exit rate	-0.47%	-0.37%
Growth rate	+0.04%	+0.04%

Table 9: Comparing Balanced Growth Paths

- Average age of young households: Mean age for households between 20 and 34, for the period 1970-1985. Data from National Intercensal Tables.
- Population share of old households: Ratio of households 35 and above to households 20 and above, for the period 1970-1985. Data from National Intercensal Tables.
- Average share of duopolist firm: Mongey 2021 calculates median two-firm revenue share using IRI Symphony data for the period 2001-2010. Markets are defined by product category, state, month. I take the two-firm revenue share, substract 0.1 (the increase in concentration from 1980-2010), and divide by 2 to get average share of duopolist firm.
- Aggregate markup: Sales weighted harmonic average of markups of firms in Compustat.
- Exit rate: Ratio of exiting firms with 20 or more employees to all firms with 20 or more employees, for the 1970-1985 period. Data from Business Dynamic Statistics.
- Growth rate: Growth rate of labor productivity for the 1970-1985 period. Data from the Federal Reserve Bank of San Francisco.
- Revenue productivity dispersion: Standard deviation of revenue productivity for firms in Compustat, for the 1970-1985 period. Revenue productivity is de-meaned by NAICS

4 digit $\times$ year

F Planning problem

F.1 Setup

The planner chooses labor inputs into production and innovation rates for each firm to maximize social welfare

$$SW_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(M_s^Y U_s^Y + M_s^O U_s^O \right) \right],$$

with

$$U_t^Y = \int \ln \left(\left[\left(C_{1jt}^Y \right)^{\frac{\rho-1}{\rho}} + \left(C_{2jt}^Y \right)^{\frac{\rho-1}{\rho}} + \int^{\mathcal{N}} C_{fjt}^Y (x)^{\frac{\rho-1}{\rho}} dx \right] \right)^{\frac{\rho}{\rho-1}} dj - L_t^Y$$

$$U_t^O = \int \ln \left(\left[(2k_{1jt})^{\frac{\theta}{\rho}} \left(C_{1jt}^O \right)^{\frac{\rho-1}{\rho}} + (2k_{1jt})^{\frac{\theta}{\rho}} \left(C_{2jt}^O \right)^{\frac{\rho-1}{\rho}} + \int^{\mathcal{N}} C_{fjt}^O (x)^{\frac{\rho-1}{\rho}} dx \right] \right)^{\frac{\rho}{\rho-1}} dj - L_t^O,$$

subject to the resource constraint

$$M_t^Y L_t^Y + M_t^O L_t^O = \int [\frac{C_{1jt}}{q_{1jt}} + \frac{C_{2jt}}{q_{2jt}} + \int \frac{C_{fjt}(x)}{q_{fjt}} dx + \frac{\gamma}{2} \left(\log \frac{1}{1-\iota_{1jt}} \right)^2 + \frac{\gamma}{2} \left(\log \frac{1}{1-\iota_{1jt}} \right)^2 + \frac{\gamma}{2} \left(\log \frac{1}{1-\iota_{jt}^{ent}} \right)^2]dj,$$

along with the law of motion for customer capital and allocation of goods among young and old households as in the market equilibrium. These two constraints are derived as follows.

From 6, we have

$$p_{ijt} = \left(C_{ijt}^Y \right)^{\frac{-1}{\rho}} \left[p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt} (x)^{1-\rho} dx \right]^{\frac{-1}{\rho}}.$$

With similar expression for $-i$ and fringe firms, we have

$$p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx = \left[\left(C_{ijt}^Y \right)^{\frac{\rho-1}{\rho}} + \left(C_{-ijt}^Y \right)^{\frac{\rho-1}{\rho}} + \int \left(C_{fjt}^Y (x) \right)^{\frac{\rho-1}{\rho}} dx \right] \left[p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho} dx \right]^{\frac{\rho-1}{\rho}}$$

$$\left[p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt} (x)^{1-\rho} dx \right]^{\frac{1}{\rho}} = \left(C_{ijt}^Y \right)^{\frac{\rho-1}{\rho}} + \left(C_{-ijt}^Y \right)^{\frac{\rho-1}{\rho}} + \int \left(C_{fjt}^Y (x) \right)^{\frac{\rho-1}{\rho}} dx.$$

Substituting back,

$$p_{ijt} = \frac{\left(C^Y_{ijt}\right)^{\frac{-1}{\rho}}}{\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}} + \left(C^Y_{-ijt}\right)^{\frac{\rho-1}{\rho}} + \int \left(C^Y_{fjt}(x)\right)^{\frac{\rho-1}{\rho}} dx},$$

so then

$$\frac{p_{ijt}C^y_{ijt}}{p_{ijt}C^y_{ijt} + p_{-ijt}C^y_{-ijt}} = \frac{\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}}}{\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}} + \left(C^Y_{-ijt}\right)^{\frac{\rho-1}{\rho}}}.$$

Similarly,

$$\frac{p_{ijt}C^o_{ijt}}{p_{ijt}C^o_{ijt} + p_{-ijt}C^o_{-ijt}} = \frac{k_{ijt}^{\frac{\theta}{\rho}}\left(C^O_{ijt}\right)^{\frac{\rho-1}{\rho}}}{k_{ijt}^{\frac{\theta}{\rho}}\left(C^O_{ijt}\right)^{\frac{\rho-1}{\rho}} + k_{-ijt}^{\frac{\theta}{\rho}}\left(C^O_{-ijt}\right)^{\frac{\rho-1}{\rho}}}.$$

Substituting into equation (5) gives the law of motion for customer capital that the planner faces:

$$\begin{aligned} k_{ijt+1} = & \frac{(1-\delta)}{\epsilon_y M_y + M_o(1-\epsilon_o)}\left(0.5\epsilon_y M_y + k_{ijt}M_o(1-\epsilon_o)\right) \\ & + \frac{\delta}{\epsilon_y M_y + M_o(1-\epsilon_o)}\left(\frac{\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}}}{\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}} + \left(C^Y_{-ijt}\right)^{\frac{\rho-1}{\rho}}}\epsilon_y M_y + \frac{k_{ijt}^{\frac{\theta}{\rho}}\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}}}{k_{ijt}^{\frac{\theta}{\rho}}\left(C^Y_{ijt}\right)^{\frac{\rho-1}{\rho}} + k_{-ijt}^{\frac{\theta}{\rho}}\left(C^Y_{-ijt}\right)^{\frac{\rho-1}{\rho}}}M_o(1-\epsilon_o)\right). \end{aligned} \tag{13}$$

For the allocation of goods among young and old households, given labor inputs $l_{ijt}, l_{-ijt}, l_{fjt}$ within a sector, market shares $s_{ijt}, s_{-ijt}, s_{fjt}$ are implicitly given by 8. We can write the demand by young household for good i as

$$C^Y_{ijt} = \frac{1}{p_{ijt}}\frac{1}{1 + \left(\frac{l_{ijt}}{s_{ijt}}\frac{s_{-ijt}}{l_{-ijt}}\frac{q_{ijt}}{q_{-ijt}}\right)^{1-\rho} + N\left(\frac{q_{ijt}}{q_{fjt}}\frac{\rho}{\rho-1}\frac{s_{ijt}}{l_{ijt}}\right)^{1-\rho}}.$$

Since $s_{ijt} = p_{ijt}C_{ijt}$, we have

$$C^Y_{ijt} = \frac{q_{ijt}l_{ijt}}{s_{ijt}}\frac{1}{1 + \left(\frac{l_{ijt}}{s_{ijt}}\frac{s_{-ijt}}{l_{-ijt}}\frac{q_{ijt}}{q_{-ijt}}\right)^{1-\rho} + N\left(\frac{q_{ijt}}{q_{fjt}}\frac{\rho}{\rho-1}\frac{s_{ijt}}{l_{ijt}}\right)^{1-\rho}}.$$

Similarly,

$$C^O_{ijt} = \frac{q_{ijt}l_{ijt}}{s_{ijt}}\frac{(2k_{ijt})^\theta}{(2k_{ijt})^\theta + (2k_{-ijt})^\theta\left(\frac{l_{ijt}}{s_{ijt}}\frac{s_{-ijt}}{l_{-ijt}}\frac{q_{ijt}}{q_{-ijt}}\right)^{1-\rho} + N\left(\frac{q_{ijt}}{q_{fjt}}\frac{\rho}{\rho-1}\frac{s_{ijt}}{l_{ijt}}\right)^{1-\rho}}.$$

F.2 Numerical solution

I solve the recursive planning problem on the BGP as in equation (12). This is done under baseline parameters as in subsection 3.3. Figure 2 shows planner and firm innovation choices over the leader-follower productivity gap, for three levels of leader customer capital. All firms in the market equilibrium under-invest in innovation compared to the planning problem.

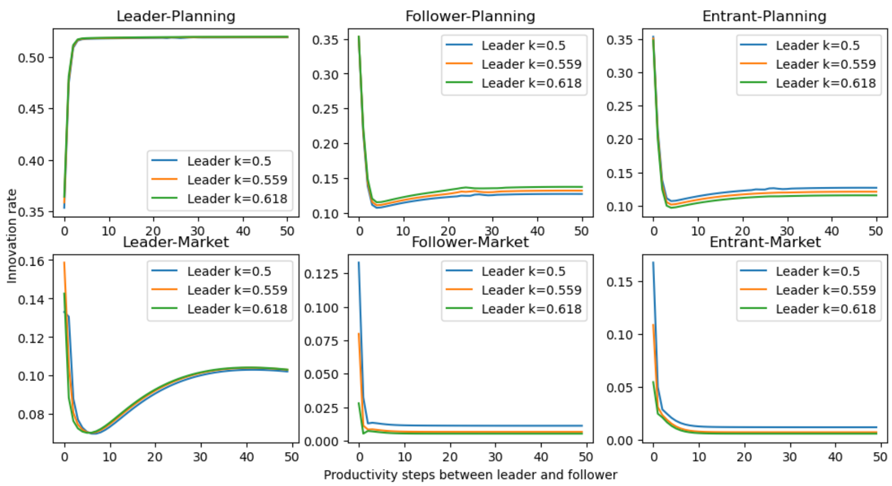

Figure 2: Innovation rates, Planner vs Market

G Additional tables and figures

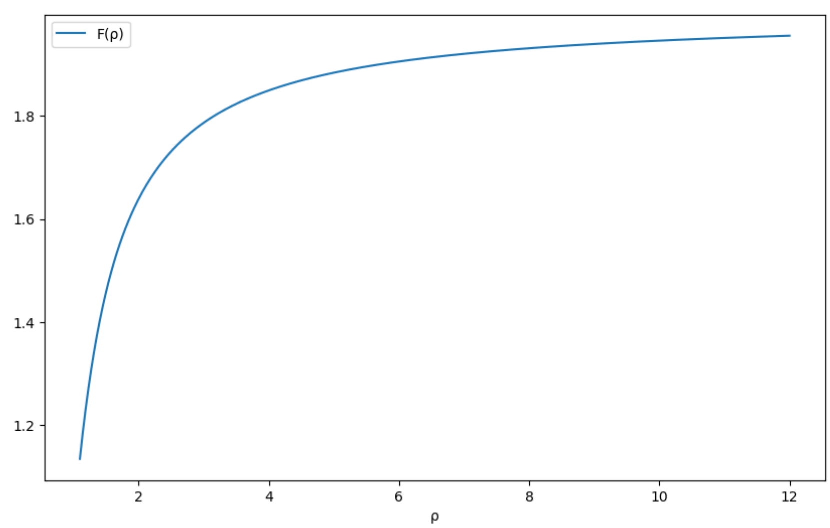

Figure 3: Graph of $F(\rho)$

Dep var	Top 90^{th}		Bottom 90^{th}		Difference	
	$(R\&D/Emp)_{jt}$	$\log(1+R\&D/Emp)_{jt}$	$(R\&D/Emp)_{jt}$	$\log(1+R\&D/Emp)_{jt}$	$(R\&D/Emp)_{jt}$	$\log(1+R\&D/Emp)_{jt}$
S_{jt}	8.01 (3.23)	10.53 (3.76)	3.49 (1.90)	1.31 (0.69)	7.73 (2.62)	13.55 (3.01)
N ind	28	28	28	28	28	28
N ind×time	229	229	260	260	221	221

Note: T-stat in parentheses. Heteroskedastic robust standard errors.

Table 10: Consumption from older households and R&D/Employment

Dep var	Top 90^{th}		Bottom 90^{th}		Difference	
	$(R\&D/Sale)_{jt}$	$\log(1+R\&D/Sale)_{jt}$	$(R\&D/Sale)_{jt}$	$\log(1+R\&D/Sale)_{jt}$	$(R\&D/Sale)_{jt}$	$\log(1+R\&D/Sale)_{jt}$
S_{jt}	7.28 (2.48)	7.55 (2.45)	4.92 (1.97)	4.95 (1.98)	7.13 (1.62)	7.76 (1.68)
N ind	28	28	28	28	28	28
N ind×time	232	232	265	265	224	224

Note: T-stat in parentheses. Heteroskedastic robust standard errors.

Table 11: Consumption from older households and R&D/Sales

Dep var	Top 90^{th}		Bottom 90^{th}		Difference	
	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$
S_{jt}	9.92 (1.86)	16.90 (3.65)	4.62 (1.03)	0.91 (0.29)	6.00 (1.20)	18.27 (2.65)
N ind	28	28	28	28	28	28
N ind×time	232	232	265	265	224	224

Note: T-stat in parentheses. Heteroskedastic robust standard errors. Industry weighted by expenditures from the CEX.

Table 12: Consumption from older households and innovation, industry weighted by expenditures