Customer capital and firm innovation

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Introduction

- Customer capital: Value from customer base through repeated transactions
- Important form of intangible capital:
  - Linked to firm valuation, sales, lower default risk
  - Large expense on advertising and sales; Affects how firm set prices >
- Paper goal: Study effect of customer capital on firm decision to innovate
  - Important for understanding trends in productivity dispersion, concentration, markups
  - Matters for innovation subsidies

This paper

- Develop model where
  - Firms innovate to reduce cost; Customer capital arise from consumption habits, where older households have stronger habits
  - Model makes predictions on how strength of customer capital affects R&D spending and productivity dispersion
  - Validate using industry age composition of demand

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  - Model makes predictions on how strength of customer capital affects R&D spending and productivity dispersion
  - Validate using industry age composition of demand
- Motivated by higher consumption persistence for older households
  - Quantify effect of aging demographics: Generates 10%-35% of observed movements in R&D spending differences, concentration, markups

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- Motivated by higher consumption persistence for older households
  - Quantify effect of aging demographics: Generates 10%-35% of observed movements in R&D spending differences, concentration, markups
- Innovation subsidies with customer capital
  - Amplified effect on concentration and markups, 2-3 times greater than without customer capital

Literature

- Customer capital:
  - Larkin (2013), Gourio and Rudanko (2014), **Foster et. al. (2016)**, Baker et. al. (2023), Afrouzi et. al. (2023) ⇒ Effect on firm innovation
- Intangibles and innovation:
  - Cavenaile and Roldan-Blanco (2020), **Cavenaile et. al. (2023)**, Shen (2023), De Ridder (2024) ⇒ Persistent customer capital + competition structure
- Accounting for aggregate trends in productivity dispersion, concentration, markups:
  - Karahan et. al. (2019), Peters and Walsh (2021), **Bornstein (2021)**, **Olmstead-Rumsey (2022)**, **Akcigit and Ates (2023)** ⇒ Complementary demand mechanism

Outline

- Simple model
- Quantitative model
- Calibration
- Empirical support
- Effect of aging demographics
- Innovation subsidies with customer capital

Simple model

Simple model

- Two period duopoly
- First period: No production. Firm $i \in \{1, 2\}$ comes in with productivity $\mathring{q}_i$, invest in R&D ι_i to increase productivity in second period
  - Second period productivity $q_i = \begin{cases} \lambda \mathring{q}_i & \text{with probability } \iota_i \\ \mathring{q}_i & \text{with probability } 1\text{-}\iota_i \end{cases}$
  - Cost of R&D: $\frac{\gamma}{2}\iota_i^2$
- Second period: Cournot competition, marginal cost $1/q_i$

Simple model

- Unit mass households, 1 unit of endowment to spend >
  - Preference: $\left(k_1^{\frac{\theta}{\rho}} c_1^{\frac{\rho-1}{\rho}} + k_2^{\frac{\theta}{\rho}} c_2^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$
    - ★ Habits/Customer capital (k_1, k_2); Habit strength θ
  - Inverse demand:

$$p_i = \frac{(k_i)^{\theta/\rho} c_i^{-1/\rho}}{(k_i)^{\theta/\rho} c_i^{\frac{\rho-1}{\rho}} + (k_{-i})^{\theta/\rho} c_{-i}^{\frac{\rho-1}{\rho}}}$$

  - More customer capital k_i raise demand & reduce demand elasticity

Firm problem

- Second period:
  - Cournot game with payoff $\pi_i = (p_i - 1/q_i)\, c_i$
  - $\Rightarrow$ Equilibrium payoffs $\pi\left(k_i/k_{-i}, q_i/q_{-i}\right)$ >
- First period:
  - Approximation of FOC

$$\iota_i \approx \frac{1}{\gamma}\left[\pi\left(k_i/k_{-i}, \lambda\mathring{q}_i/\mathring{q}_{-i}\right) - \pi\left(k_i/k_{-i}, \mathring{q}_i/\mathring{q}_{-i}\right)\right]$$

Innovation rates and customer capital

Proposition

An increase in $(k_i/k_{-i})^{\theta}$ *increases* ι_i *iff* $F(\rho)$

$$\underbrace{\left(\frac{k_i}{k_{-i}}\right)^{\frac{\theta}{\rho}}\left(\frac{\mathring{q}_i}{\mathring{q}_{-i}}\right)^{\frac{\rho-1}{\rho}}}_{\text{Relative revenue productivity}} < \underbrace{F(\rho)}_{\text{Known function}}$$

- Customer capital have opposing effects on innovation
  - Higher demand ⇒ Produce more ⇒ Larger cost reduction from innovation
  - Lower elasticity ⇒ Restrict supply for markups ⇒ Lower innovation
- Total effect depends on relative revenue productivity

Takeaway - effect of customer capital

- Relative revenue productivity $< 1 < F(\rho)$ for follower $\Rightarrow$ innovation moves with customer capital
- For leader:
  - With reasonable ρ, $\log F(\rho) \approx 0.66$; Std of log revenue prod. ≈ 0.28 for public firms
  - $\Rightarrow$ innovation moves with customer capital
- With stronger habits $(\theta \uparrow)$, innovation increase for leader $\left((k_i/k_{-i})^{\theta} \uparrow\right)$ and decrease for follower $\left((k_{-i}/k_i)^{\theta} \downarrow\right)$
  - $\Rightarrow$ Leader increase productivity gap $((q_i/q_{-i}) \uparrow)$, capture more market share, charge higher markups

Takeaway - what the static model misses

- With dynamics and endogenous customer capital,
  - Leader produce more $\Rightarrow$ accumulate more k_i $\Rightarrow$ increase innovation $\Rightarrow$ larger $\frac{q_i}{q_{-i}}$ $\Rightarrow$ leader produce more
  - Amplified movements in productivity gap
  - Amplified effect of innovation subsidy on concentration

Quantitative model

Quantitative model

- Dynamic duopoly, continuum of industries
- Two types of households, young and old >
- Habit evolution: Accumulated past expenditure of average old household >
- Mass of fringe firms in addition to two dominant firms >
- Follower have additional chance to catch up >
- Entrant replacing follower > calib

Households

- Unit mass. Young $\rightarrow$ old with probability ϵ^Y. Old $\rightarrow$ dropout with probability ϵ^O; replaced by young household
  - Mass of young and old: M^Y, M^O
- Consume goods by duopolist + continuum of fringe of mass $\mathcal{N}$
- Preferences

$$U_t^a = \ln C_t^a - L_t^a$$

C_t^a: nested CES, outer nest elasticity of 1, inner nest elasticity of ρ > back

Households - Demand

- Firm i, sector j, time t

- Household demand for good ijt alternative back:

$$C_{ijt}^{Y} = \frac{p_{ijt}^{-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}\left(x\right)^{1-\rho} dx}$$

$$C_{ijt}^{O} = \frac{\left(k_{ijt}\right)^{\theta} p_{ijt}^{-\rho}}{\left(k_{ijt}\right)^{\theta} p_{-ijt}^{1-\rho} + \left(k_{-ijt}\right)^{\theta} p_{-ijt}^{1-\rho} + \left(0.5\right)^{\theta} \int^{\mathcal{N}} p_{fjt}\left(x\right)^{1-\rho} dx}$$

Households - Demand

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$$C_{ijt}^{Y} = \frac{p_{ijt}^{-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}$$

$$C_{ijt}^{O} = \frac{(k_{ijt})^{\theta}\, p_{ijt}^{-\rho}}{(k_{ijt})^{\theta}\, p_{-ijt}^{1-\rho} + (k_{-ijt})^{\theta}\, p_{-ijt}^{1-\rho} + (0.5)^{\theta} \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx}$$

- Habits k_{ijt} affect old consumption, increases demand, decreases elasticity

Households - Habits

- Habits evolution back

$$[\text{Stock of habits tomorrow}] = (1-\delta)\,[\text{Stock of habits today}] + \delta\,[\text{Relative expenditures today}]$$

Households - Habits

- Habits evolution back

$$k_{ijt+1} = (1-\delta)\overbrace{\frac{0.5\epsilon^Y M^Y + k_{ijt} M^O \left(1-\epsilon^O\right)}{\epsilon^Y M^Y + M^O\left(1-\epsilon^O\right)}}^{\text{Stock of habits today}} + \underbrace{\delta\left[\frac{p_{ijt}C^Y_{ijt}}{p_{ijt}C^Y_{ijt} + p_{-ijt}C^Y_{-ijt}}\epsilon^Y M^Y + \frac{p_{ijt}C^O_{ijt}}{p_{ijt}C^O_{ijt} + p_{-ijt}C^O_{-ijt}} M^O\left(1-\epsilon^O\right)\right]\frac{1}{\epsilon^Y M^Y + M^O\left(1-\epsilon^O\right)}}_{\text{Relative expenditures today}}$$

  - Average of young households turning old tomorrow and old households alive tomorrow

Households - Habits

- Habits evolution back

$$k_{ijt+1} = (1-\delta)\frac{0.5\epsilon^Y M^Y + k_{ijt} M^O\left(1-\epsilon^O\right)}{\epsilon^Y M^Y + M^O\left(1-\epsilon^O\right)} +$$

$$\delta\left[\frac{p_{ijt}C^Y_{ijt}}{p_{ijt}C^Y_{ijt} + p_{-ijt}C^Y_{-ijt}}\epsilon^Y M^Y + \frac{p_{ijt}C^O_{ijt}}{p_{ijt}C^O_{ijt} + p_{-ijt}C^O_{-ijt}} M^O\left(1-\epsilon^O\right)\right]\frac{1}{\epsilon^Y M^Y + M^O\left(1-\epsilon^O\right)}$$

- External habits, accumulate from past expenditure of other old households
  - Average of young households turning old tomorrow and old households alive tomorrow
  - Representative old household consume more of good today → like it more → consume more tomorrow with less consideration for prices
  - Customer capital for the firms

Firms

- Duopolists compete in quantities back
  - For variable x, denote leader with $\overline{x}$ and follower with $\underline{x}$
- Duopolist invest in R&D to increase productivity next period:
  - Production: $Y_{ijt} = q_{ijt} l_{ijt}$
  - Leader productivity: $\overline{q}_{jt+1} = \overline{D}_{jt} \lambda \overline{q}_{jt} + \left(1 - \overline{D}_{jt}\right) \overline{q}_{jt}$; $\overline{D}_{jt} = 1$ with prob. $\overline{\iota}_{jt}$

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  - Follower productivity: $\underline{q}_{jt+1} = \underline{D}_{jt} \left(1 - \Phi\right) \lambda \underline{q}_{jt} + \underbrace{\underline{D}_{jt} \Phi \overline{q}_{jt}}_{\text{Closing the gap}} + \left(1 - \underline{D}_{jt}\right) \underline{q}_{jt}$;

    $\underline{D}_{jt} = 1$ with prob. $\underline{\iota}_{jt}$; $\Phi = 1$ with prob. ϕ
  - Fringe productivity: $q_{fjt} = \overline{q}_{jt}^{\alpha} \underline{q}_{jt}^{1-\alpha}$
- Cost of R&D: $\frac{\gamma}{2} \left(\log \left(\frac{1}{1 - \iota_{ijt}} \right) \right)^2$

Firms - Entrants

- Prospective entrant in each sector each period
- Conducts R&D to innovate on the follower's technology
- If productivity higher than the follower's (ie successful innovation), replace the follower
  - Inherit follower customer capital stock back

Firms

- Define: $m = (\log q - \log q_-) / \log \lambda$; $\pi = p * Y - l$; $\mathcal{R}$ indicator if firm is replaced by entrant
- Duopolist solve

$$v(k, k_-, m) = \max_{l,\iota} \pi(l, l_-, k, k_-, m) - \frac{\gamma}{2}\left(\log\left(\frac{1}{1-\iota}\right)\right)^2 + \beta E_{m',\mathcal{R}}\left[v(k', k'_-, m')(1-\mathcal{R})\right]$$

  - Choice of l affects π today and k' tomorrow
  - Choice of ι affects $m', \mathcal{R}$ tomorrow eqm

Calibration

Model parameterization

- Model calibrated to match moments from US in 1980

Param	Description	Value	Param	Description	Value
	External			Internal	
β	Discount rate	0.99	λ	Growth step size	1.065
ϵ^Y	Prob. of turning old	0.0357	$\mathcal{N}$	Mass of fringe	6.5
ϵ^O	Prob. of death	0.0192	α	Fringe productivity weight	0.808
ρ	Sectoral elas. of substitution	10	γ	Cost of R&D	4.05
δ	Depreciation of consumer habit	0.0133	ϕ	Prob of closing gap, upon success	0.212
			θ	Strength of consumer habit	2.2

Model moments

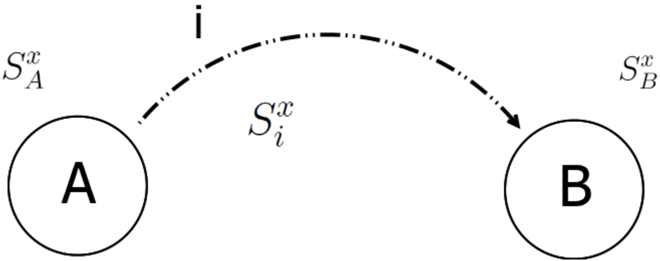
Moment	Model	Target	Source
Revenue productivity dispersion	0.203	0.20	Compustat
Relative change in market share after price change	0.677	0.68	Bronnenberg et. al. (2012)
Aggregate markups	1.281	1.28	Compustat
Growth rate	2.22%	2.2%	SF Fed
Mean market share	0.265	0.26	Mongey (2021)
Entry rate	1.87%	1.82%	BDS

Disciplining habit parameters

- Markets A, B with goods x, y; Market share S_A^x, S_B^x

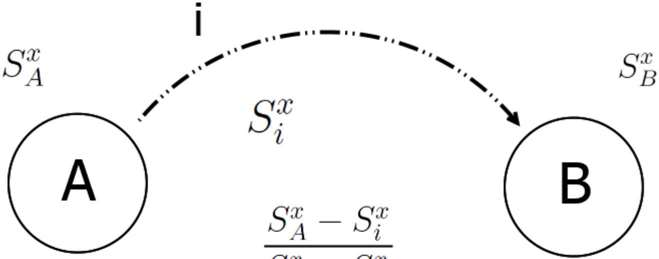
Disciplining habit parameters

- i moves $A \rightarrow B$, track i's expenditure share S_{it}^{x}
- Before move, $S_{i0}^{x} = S_{A}^{x}$; Over time, $\lim_{t \rightarrow \infty} S_{it}^{x} = S_{B}^{x}$



Disciplining habit parameters

- $\frac{S_A^x - S_{i1}^x}{S_A^x - S_B^x}$ - period after move - informs strength of habits: Closer to 0 $\Rightarrow$ stronger habits
- How fast $\frac{S_A^x - S_{it}^x}{S_A^x - S_B^x}$ converges to 1 over time informs persistence

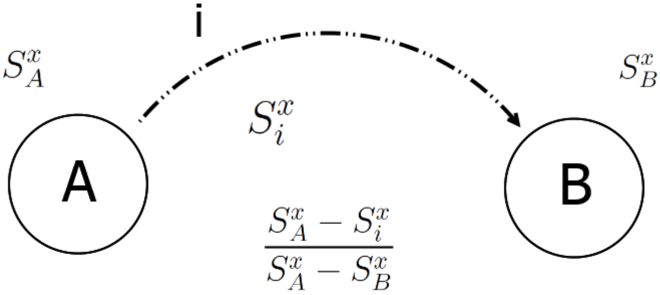


S_A^x S_i^x S_B^x

$$\frac{S_A^x - S_i^x}{S_A^x - S_B^x}$$

Disciplining habit parameters - Market analog

- Start at long run A. Market conditions (ie price) changes s.t. new long run is B
- $\frac{S_A^x - S_{i1}^x}{S_A^x - S_B^x}$ informs strength of habits



Disciplining habit parameters - Implementation

- Initial state A:
  - Same productivity across 2 firms; set $k_A^x > 0.5$ at long run level; calculate share S_A^x
- Change market condition:
  - Firm x innovates; with $k^x = k_A^x$, obtain new price under eqm policy rules
  - Hold price and productivity constant, get new long run shares S_B^x and track evolution of shares S_{it}^x
- Calibrate strength of habits so that $\frac{S_A^x - S_{i1}^x}{S_A^x - S_B^x}$ matches target

Empirical support

Empirical support

- Run analysis at industry level:
  - Relationship between customer capital and innovation efforts across leaders/followers
- Proxy for strength of customer capital using expenditure share by older households in industry >
- Project difference in innovation between leaders and followers on proxy >
- Compare to regression on model simulated data >

Proxy

- Expenditure share by older households proxy:
  - Consumption significantly more persistent for households age 35 and older (Bornstein 2021) $\sim$ larger customer capital effect
  - Proxy for strength of customer capital at the industry level back

Industry panel

- Data:
  - Panel of industries, 1990 to 2019
  - R&D from Compustat, consumption share from Consumer Expenditure Survey
  - Restrict to industries with high percentage of output used as final goods
  - Take average observations in bins of 3 years

Age composition of demand

- Comovements of older households expenditure share with R&D spending difference between leader and follower ex
- Regressions:

$$Y_{jt} = \gamma S_{jt} + \boldsymbol{\eta} \boldsymbol{A}_{jt} + \delta_j + \nu_t + \varepsilon_{jt}$$

  - Y_{jt}: Difference in R&D spending of top 90^{th} productive firms and other firms in industry, standardized
  - S_{jt}: Share of expenditures by households age 35 and over
  - $\boldsymbol{A}_{jt}$: Controls: Standard deviation of log revenue productivity; Total household expenditure on industry

Age composition of demand

Dep var	$R\&D_{jt}$	$\log(1 + R\&D)_{jt}$
S_{jt}	10.07	11.96
	(1.72)	(2.36)
N ind	28	28
N ind$\times$time	224	224

T-stat in parentheses. Heteroskedastic robust standard errors.

back sum more disp rsize weighted patents

Age composition of demand

- Larger share of expenditure from older households:
  - Larger difference in innovation between top and non-top firms
  - Consistent with model

Quant. model comparison

- Simulate model along transition path of ϵ^O to match fraction of older households from 1960 to 2060
- Run regression on simulated sectors from 1990 to 2019:
  - Project R&D spending difference between high and low productivity firms in sector, standardized, on share of expenditure by older households back

Quant. model comparison

	Simulated	Empirical	
R&D	6.60	11.89	10.07
		$(-0.52, 24.30)$	$(-1.45, 21.59)$
$\log(1 + \text{R\&D})$	6.62	10.41	11.96
		$(3.34, 17.62)$	$(1.97, 21.95)$
FE	Ind	Ind	Ind, Time

95% confidence interval in parentheses

Quantifying effect of aging demographics

Aging demographics - Comparing BGPs

- Decrease ϵ^O to match fraction of older households in 2020

	Model		Data
Fraction of older households	0.65	0.72	
R&D divergence	0.0171	+0.115 std	+0.524 std
Revenue productivity dispersion	0.203	+0.053	+0.113
Aggregate markups	1.281	+0.074	+0.11
Mean market share	0.265	+0.032	+0.05
Entry/Exit rate	1.87%	-0.47%	-0.51%
Growth rate	2.22%	+0.04%	-0.36%

Aging demographics - Along the transition

- Transition along observed and predicted path of fraction of older households from 1960 to 2060, starting from BGP Fig Pol

Year	Model		Data
	1980	2020	1980-2020 change
R&D divergence	0.0178	+0.151 std	+0.524 std
Revenue productivity dispersion	0.203	+0.01	+0.113
Aggregate markups	1.28	+0.02	+0.11
Mean market share	0.264	+0.017	+0.05
Entry/Exit rate	1.86%	-0.51%	-0.51%

Aging demographics - Along the transition

Figure: Evolution of measures along the transition back

Effect of aging demographics

- More weight on older households in demand $\rightarrow$ stronger effect of customer capital for firms
  - Leaders enjoy larger pool of customers with strong habits to sell to $\sim$ more k for leaders
  - Followers sell less as pool of customers without habits for leader's good shrink $\sim$ less k for followers
  - $\Rightarrow$ Larger difference in innovation $\Rightarrow$ Leaders widen productivity gap over followers
  - $\Rightarrow$ Increased concentration, increased markups

Effect of aging demographics

- Aging demographics can account for sizable portion of trends in revenue productivity dispersion, aggregate markups, concentration
  - Comparing BGPs, changes are around 50% of observed trends
  - Over the transition, changes are around 10%-35% of observed trends; predicted to keep increasing >

Innovation subsidies with customer capital

Policy motivation

- Equilibrium inefficient:
  - Low production: Firm charge markups
  - Low innovation: Firm profit gains $<$ Social gains
- Government can improve on equilibrium through mix of production subsidy and innovation subsidy
- Consider subsidy to entry and incumbent R&D
  - Compare to BGP without customer capital

Subsidy to R&D

10% subsidy to R&D cost	With customer capital	Without customer capital
Revenue productivity dispersion	+9.38%	+1.66%
Mean market share	+2.84%	+1.11%
Aggregate Markups	+1.32%	+0.39%
Entry rate	-5.55%	+0.41%
Growth rate	+8.64%	+8.62%
Welfare (CE)	+4.24%	+4.15%

*Percentage deviation from baseline

Subsidy to R&D

- Leader and follower innovation increase proportionally $\Rightarrow$ innovation difference increase $\Rightarrow$ widen productivity gap

- With customer capital, leader produce more $\Rightarrow$ build more customer capital $\Rightarrow$ innovate more $\Rightarrow$ further widen productivity gap entry

Conclusion

- Customer capital affects firm innovation and industry concentration as consequence
- Changes in customer capital, associated with aging demographics, generates sizable portion of aggregate trends in productivity dispersion, concentration, markups
- Effect of innovation subsidies on market structure amplified with customer capital
  - Additional consideration for policy makers when designing policies

Subsidy to entry

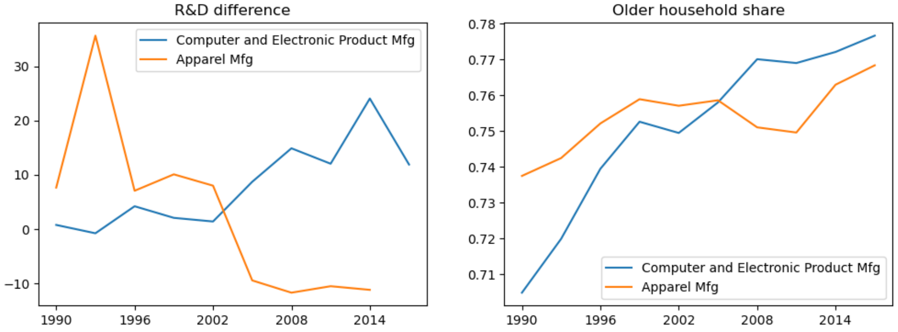
10% subsidy to entry cost	With customer capital	Without customer capital
Revenue productivity dispersion	+0.71%	-1.09%
Mean market share	-1.04%	-0.45%
Aggregate Markups	-0.46%	-0.21%
Entry rate	+8.20%	+6.52%
Growth rate	+0.08%	+0.21%
Welfare (CE)	+0.13%	+0.12%

*Percentage deviation from baseline

Subsidy to entry

- Entry subsidy decrease productivity dispersion, concentration, markups rnd
  - Entrant innovate on follower's tech. $\Rightarrow$ higher entry reduce productivity gap
  - Larger effect with customer capital: Lower productivity gap $\Rightarrow$ lower k for leaders $\Rightarrow$ lower innovation difference $\Rightarrow$ lower productivity gap

Example of 2 industries



back

Age composition of demand - scaled

Dep var	$(R\&D/Emp)$	$\log(1+R\&D)/Emp$	$(R\&D/Asset)$	$\log(1+R\&D)/Asset$	$(R\&D/Sale)$	$\log(1+R\&D)/Sale$
S	9.62	6.44	7.07	6.74	8.45	8.67
	(2.57)	(1.71)	(1.61)	(1.50)	(1.61)	(1.63)
N ind	28	28	28	28	28	28
N ind×time	221	221	224	224	224	224

T-stat in parentheses. Heteroskedastic robust standard errors. back

Age composition of demand - weighted

Dep var	$R\&D_{jt}$	$\log(1+R\&D)_{jt}$
S_{jt}	4.86	14.51
	(0.80)	(2.20)
N ind	28	28
N ind×time	224	224

T-stat in parentheses. Heteroskedastic robust standard errors. back

Age composition of demand - more

Dep var	Top 90^{th}		Bottom 90^{th}	
	$R\&D_{jt}$	$\log(1 + R\&D)_{jt}$	$R\&D_{jt}$	$\log(1 + R\&D)_{jt}$
S_{jt}	7.96	10.49	-0.38	-1.41
	(1.74)	(2.89)	(-0.16)	(-0.62)
N ind	28	28	28	28
N ind$\times$time	232	232	265	265

T-stat in parentheses. Heteroskedastic robust standard errors. back

Age composition of demand - patents

Dep var	$\log(1+CW)$	$\log(1+CW)/Emp$	$\log(1+CW)/Asset$	$\log(1+CW)/Sale$
S_{jt}	-3.07	10.53	15.03	14.67
	(-0.59)	(1.72)	(1.66)	(1.73)
N ind	28	28	28	28
N ind×time	235	235	235	235

CW: Citation weighted patent count, calculated for each firm i in time t as $\sum_{p \in P_{it}} 1 + \frac{C_p}{\bar{C}_t}$. T-stat in parentheses.

Heteroskedastic robust standard errors. back

Effect of consumption shares on dispersion

- Changes in dispersion affected by gap in innovation rate between leader and follower

$$\Delta Disp_{jt+1} = (\iota_{ijt} - \iota_{-ijt}) \ln \lambda$$

- Regression for dispersion:

$$\Delta Disp_{jt+1} = \beta S_{jt} + \boldsymbol{\theta} \boldsymbol{D}_{jt} + \alpha_j + \eta_t + \epsilon_{jt}$$

- $\Delta Disp_{jt+1}$: Change in the standard deviation of log revenue productivity
- S_{jt}: share of expenditures by households age 35 and over
- $\boldsymbol{D}_{jt}$: Controls: Total household expenditure on industry

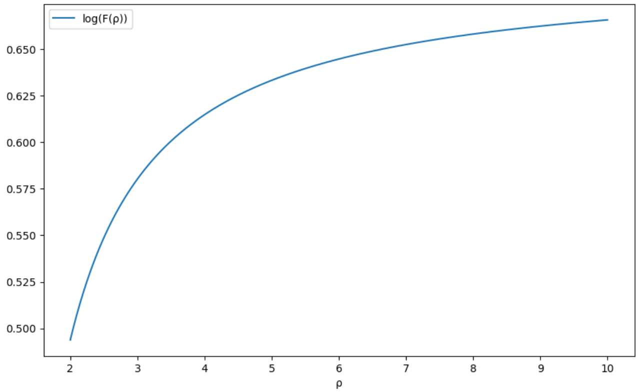
Effect of consumption shares on dispersion

Dep var	$\Delta Disp_{jt+1}$
S_{jt}	0.75
	(2.59)
N Ind	28
N Ind $\times$ Time	258

back

Log $F(\rho)$

back



Recursive Equilibrium

- Household policies, firm policies, firm value, and law of motion where
  - Household demand is optimal, given firm policies
  - Given household demand and competitor's policies, firm value solves the firm's Bellman and policies are consistent with maximization
  - Law of motion consistent with firm policies back

Households

back

- Budget:

$$P_t^a C_t^a + P_t^A A_{t+1}^a = L_t^a + \left(P_t^A + d_t\right) A_t^a$$

- Aggregator: $C_t^a = \exp\left[\int \ln C_{jt}^a dj\right]$

$$C_{jt}^Y = \left(0.5^{\frac{-\theta}{\rho}} \left[0.5^{\frac{\theta}{\rho}} \left(C_{1jt}^Y\right)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \left(C_{2jt}^Y\right)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \int^{\mathcal{N}} C_{fjt}^Y(x)^{\frac{\rho-1}{\rho}} dx\right]\right)^{\frac{\rho}{\rho-1}}$$

$$C_{jt}^O = \left(0.5^{\frac{-\theta}{\rho}} \left[k_{1jt}^{\frac{\theta}{\rho}} \left(C_{1jt}^O\right)^{\frac{\rho-1}{\rho}} + k_{2jt}^{\frac{\theta}{\rho}} \left(C_{2jt}^O\right)^{\frac{\rho-1}{\rho}} + 0.5^{\frac{\theta}{\rho}} \int^{\mathcal{N}} C_{fjt}^O(x)^{\frac{\rho-1}{\rho}} dx\right]\right)^{\frac{\rho}{\rho-1}}$$

Dispersion trend

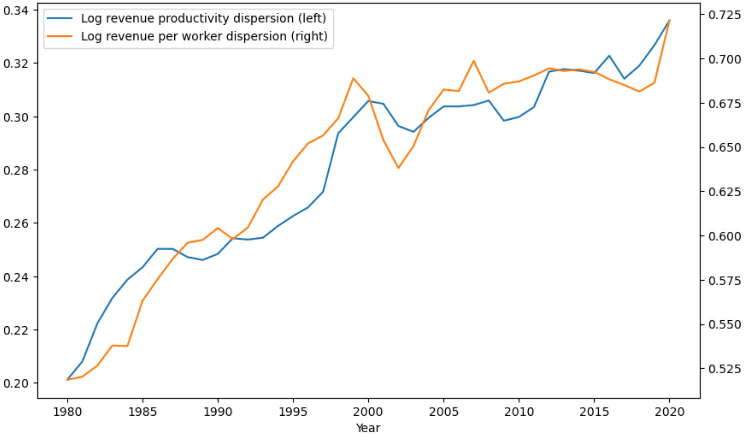
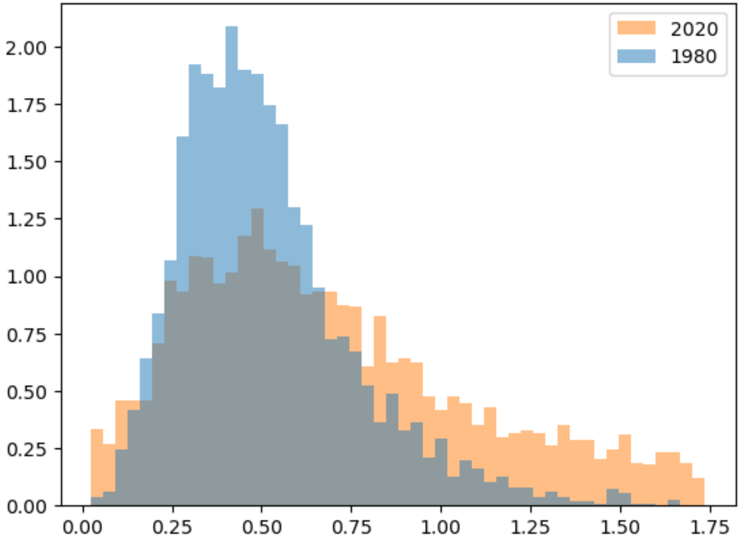


Figure: Between firm TFPR std and Sale/employment std back

Dispersion cross-section



R&D divergence

- Increasing divergence in R&D investment between more productive firms and less productive firms within industry

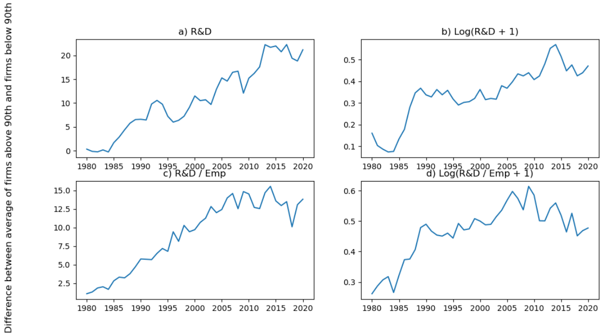


Figure: Difference of mean R&D spending between upper and lower firm quantiles by revenue productivity back

Age expenditure trend

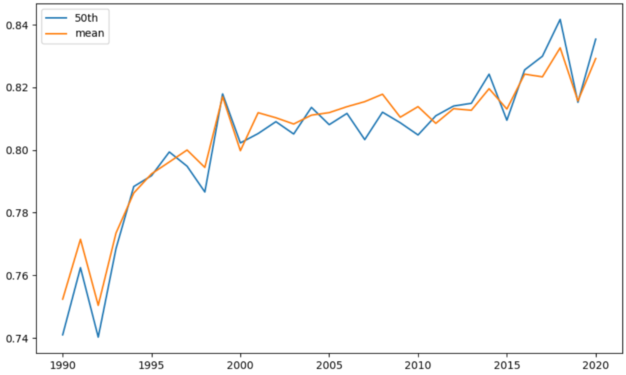


Figure: Share of expenditure of 36yo and above households, 3 digit NAICS back

Firms

- Firm profits: $\pi_{ijt} = p_{ijt} C_{ijt} - \frac{C_{ijt}}{q_{ijt}} \equiv s_{ijt} - l_{ijt}$, where s_{ijt} is implicitly defined by

$$s_{ijt} = \frac{p_{ijt}^{1-\rho}}{p_{ijt}^{1-\rho} + p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx} M_y + \frac{(2k_{ijt})^{\theta}\, p_{-ijt}^{1-\rho}}{(2k_{ijt})^{\theta}\, p_{-ijt}^{1-\rho} + (2k_{-ijt})^{\theta}\, p_{-ijt}^{1-\rho} + \int^{\mathcal{N}} p_{fjt}(x)^{1-\rho}\, dx} M_o$$

$$\frac{p_{-ijt}}{p_{ijt}} = \frac{l_{ijt}}{l_{-ijt}} \frac{s_{-ijt}}{s_{ijt}} \frac{q_{ijt}}{q_{-ijt}}; \quad \frac{p_{ijt}}{p_{fjt}} = \left(\frac{1}{q_{fjt}} \frac{\rho}{\rho - 1} \right)^{-1} \frac{s_{ijt} l_{ijt}^{-1}}{q_{ijt}}$$

back

Discrete choice demand setup

- Sectoral preference:

$$C_{jt}^{Y} = \max_{ijt} \left[\exp\left(\frac{1}{\rho - 1} \epsilon_{ijt} \right) C_{ijt}^{a} \right]$$

$$C_{jt}^{O}\left(\{k_{ijt}\}\right) = \max_{ijt} \left[\exp\left(\frac{1}{\rho - 1} \left[\epsilon_{ijt} + \theta \ln\left(2k_{ijt}\right)\right] \right) C_{ijt}^{a} \right]$$

Discrete choice demand setup

- Good chosen to solve:
  - For young:
  $$\max_{ijt} -(\rho - 1)\ln p_{ijt} + \epsilon_{ijt}$$
  - For old:
  $$\max_{ijt} -(\rho - 1)\ln p_{ijt} + \epsilon_{ijt} + \theta \ln (2k_{ijt})$$

  with ϵ_{ijt} iid Type I Extreme Value

back

Summary stats

	Difference		
	S	R&D	log(1+R&D)
Std, controlling for ind and time	0.014	21.97	0.83

back

Simple model - Discrete choice

- Unit mass households, 1 unit of endowment to spend back
  - Preference: $\exp\left(\frac{1}{\rho-1}\left[\epsilon_i^h + \theta \ln\left(k_i\right)\right]\right) c_i$ for $i \in \{1,2\}$ with ϵ_i^h iid extreme value shocks
    - ★ Habits/Customer capital (k_1, k_2); Habit strength θ
  - Household choice: $i^h = \arg\max_i (1-\rho)\log p_i + \theta \log k_i + \epsilon_i^h$
  - Choice probability of choosing i for household h:

$$\frac{k_i^\theta p_i^{1-\rho}}{k_i^\theta p_i^{1-\rho} + k_{-i}^\theta p_{-i}^{1-\rho}}$$

Eqm Profits

$$\pi\left(k_i/k_{-i}, q_i/q_{-i}\right) = \frac{\left(\frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}}\left(\frac{q_i}{q_{-i}}\right)^{(\rho-1)/\rho} + \frac{1}{\rho}\right)\frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}}\left(\frac{q_i}{q_{-i}}\right)^{(\rho-1)/\rho}}{\left[1 + \frac{k_i^{\theta/\rho}}{k_{-i}^{\theta/\rho}}\left(\frac{q_i}{q_{-i}}\right)^{(\rho-1)/\rho}\right]^2}$$

back

Relevance of Customer Capital

- Brand capital 6-25% of firm value (Belo et. al. 2022)
- Product familiarity associated with lower default risk (Larkin 2013)
- Differences in customer base accounts for 80% sale variances (Einav et. al. 2021, Afrouzi et. al. 2023)
- Firm spending on advertising, sales expenditures, customer service around 2/3 of physical capital spending (He et. al. 2024)
- Firms stabilize prices to maintain long-run customer relationship (Blinder et. al. 1998, Fabiani et. al. 2006)
- New firm formation declines when consumer inertia rises (Bornstein 2021) back