Concentration, Markups, and Output Volatility

Duong Dang

September, 2024

Abstract

This paper explores how a rise in industry concentration could lead to a decline in aggregate output volatility. I build a menu cost model with a continuum of sectors, where each sector has two dominant firms and a fringe of monopolistically competitive firms. When faced with movements in marginal costs from TFP shocks, dominant firms pass less of the changes through to prices compared to monopolistically competitive firms. This arises from strategic complementarities in pricing. Dominant firms want to price close to their rivals, generating feedback that amplifies the effect of menu costs on limiting movements in prices. Less movements in prices imply more stable demand, hence lower output volatility, so that the effects of TFP movements are dampened for dominant firms. With higher concentration, the composition of aggregate economic activity is shifted towards dominant firms, dampening the effects of TFP movements on aggregate output volatility. In the calibrated model, a 10 percentage point increase in concentration leads to a 0.017 percentage point decrease in the standard deviation of output volatility – around 2% of the decrease in volatility experienced from the Great Moderation.

1 Intro

The US economy after the 1980s saw a dip in aggregate output volatility. The standard deviation (std) of detrended GDP fell from 1.77% for the 1955-1984 period to 0.97% for the 1985-2019 period. Figure **1** plots a rolling 120 quarter window of GDP std, which shows the decline in volatility being concentrated around the 1980s-1990s period. Understanding the cause of this "Great Moderation" is important for knowing how to maintain this low volatility, and whether this period of low volatility will persist.

From the early 1980s, there has been a rise in concentration across industry sectors (Autor, Dorn, Katz, Patterson, and Van Reenen 2020), as well as a rise in markups across sectors and in the aggregate (DeLoecker, Eeckhout, and Unger 2020), hinting at increased market power. This is indicative of more economic activity being shifted towards larger firms within an industry. This paper explores a link between these two trends: how higher concentration leads to a decrease in output volatility.

The intuition is as follows. Empirically, cost pass through is lower for larger firms: given a positive supply (cost reducing) shock, large firms decrease their price less than small firms. Their markups, the ratio of price to marginal costs, then covaries more strongly with positive supply shocks. Markups drive a wedge between input supply and demand. Higher markups decrease input quantities, hence output quantities. This goes against the effects of a positive supply shock. An increase in concentration implies two things: that large firms become larger, which changes their cost pass through behaviour; and that large firms matter more in determining aggregate markups - a composition effect. Aggregate markup then covaries more strongly with positive supply shocks, in particular TFP, following an increase in concentration. This dampens the effect of TFP movements on output volatility.

To study the mechanism that generates differing responses of markups between large and small firms, and to quantify the effect of increasing concentration, I build a nested CES model in the spirit of Atkeson and Burstein (2008). There is a continuum of sectors, and in each sector there are two positive mass firms and a continuum of zero mass firms, which I refer to as respectively as large and small firms. The model has two features that generate differing responses of markups. First is differing exposure, where large firms' productivity changes more than small firms' following an aggregate TFP shock. Here, an increase in TFP makes large firms relatively more productive compared to small firms, inducing them to decrease their markups. This type of effect has been studied in the exchange rate pass through literature (for example see Amiti, Itshkoki, and Konings 2019).

The second feature is adjustment frictions. Large firms face strategic complementarities that amplifies adjustment rigidities, a channel more recently studied in Mongey (2021) and

Wang and Werning (2020). For large firms, a higher price set by their rival incentivizes the firm to raise their price as well. Consider an increase in productivity, which leads to a rise in markups. Adjustment frictions disincentivize the rival from re-adjusting markups, keeping markups high. Strategic complementarities imply that the firm would want to keep its own markups high, so that markups covary more strongly with TFP. With the two channels, a 10 percentage points increase in industry concentration, where large firms produce 10 percentage points more of aggregate output, results in a 0.017 percentage point decrease in aggregate output volatility, around 2% of the 0.8 percentage point decrease associated with the Great Moderation. For the level of concentration that the data suggests, most of this decrease in aggregate output volatility is driven by differing exposures.

This paper contributes to a literature that studies the cause of the Great Moderation. There are two main hypothesis for the cause: Good luck, where the size of shocks that hit the economy has decreased; and Good policies, where fiscal and monetary authority now responds more strongly to shocks to stabilize the economy. Stock and Watson (2002) provide a review of these hypotheses, and estimates a reduced form VAR to gauge their contribution to the decrease in output volatility. More recently, Carvalho and Gabaix (2013) look at aggregate shock as arising from sectoral shock. They argue that the decrease in the volatility of TFP over the period of the Great Moderation can be attributed to a sectoral shift in the economy, from volatile manufacturing to less volatile services. They also note how the movement in TFP volatility can be tracked by the volatility of the largest (top 100) firms in the economy. Here I study another mechanism whereby firms' endogenous response to shocks lead to a decrease in output volatility.

This paper contributes to a literature that explores concentration and market power. Autor et. al. (2020) document an increase in concentration across the US economy. They provide evidence that the rise of large firms contributes to the decline in the labour share. DeLoecker, Eeckhout,and Unger (2020) use data on publicly listed firms to estimate markups. From the 1980s, aggregate markups have increased considerable. With additional evidence on profits, they argue that market power has increased, and provide a discussion on the possible consequences. Here I take as given the rise of concentration and market power, and explore another possible consequence of this rise.

This paper draws on evidence and mechanism from the exchange rate pass through literature. The literature documents a large heterogeneity in the pass through of exchange rate shocks onto prices (for example, see Burnstein and Gopinath 2014). Auer and Schoenle (2016) estimate, using US import price micro data, that pass through take on an U shape in market share: The pass through is high for both small and very large firms, and is lower for medium sized firms. Conversely, firms' responses to their competitors' prices take on an inverse U shape in market share. This is indicative of strategic complementarity in price setting, where a firm's decision is influenced by both its own effect on its sector as well as its rivals' effects on the sector. The literature largely employs static mechanisms that generate variable cost pass through. I incorporate into my model one of these mechanism where firms have differing exposure to the aggregate shock, as used in Amiti et. al. (2019).

This paper draws on the mechanism of Mongey (2021) and Wang and Werning (2020), where by strategic complentarities increase the rigidity generated by adjustment frictions. Both works study monetary non-neutrality, whereby strategic complementarities can amplify the effect of monetary policy. Mongey (2021) compares a menu cost model with duopolists to the one with monopolistically competitive firms (ie Golosov and Lucas 2007). When firms have mass, strategic complimentarities in price setting lead to more sluggish responses in price adjustment following a monetary shock. The result is lower inflation compared to only monopolistic competition, hence a higher impact of monetary policy on output. Wang and Werning (2020) provides an analytical framework that allows for an arbitrary number of symmetric positive mass firms. While the framework I employ here only has two positive mass firms, I can vary the degree of concentration and study the outcomes under varying levels of strategic interactions.

2 Empirical evidence

2.1 Data

To assess the intuition laid out above, I require estimates of markups at the micro and aggregate level, along with estimates of TFP. For markups, I use the estimates from DeLoecker, Eeckhout, and Unger (2020). The data is Compustat annual, a yearly panel of publicly traded firms from 1950 to 2016. Individual firm markups are estimated as follows. A firm solving its cost minimization problem will equate the price of any unconstrained input to the corresponding marginal product: $P_t^V = \lambda_{it} \frac{\partial Q(\cdot)}{\partial V_{it}}$. Here P_t^V is the price of variable input V, $Q(.)$ is the production function, and λ_{it} is the Lagrangian multiplier on the output constraint. Noting that this multiplier is a direct measure of the marginal cost, the above can be arranged to give the markup

$$\mu_{it} := \frac{P_{it}}{\lambda_{it}} = \frac{P_{it} Q_{it}}{P_t^V V_{it}} \theta_{it}^v$$

where $\theta_{it}^v := \frac{\partial Q(\cdot)}{\partial V_{it}} \frac{V_{it}}{Q_{it}}$ is the output elasticity of the variable input. The markup is the inverse of the revenue share of variable input, times its output elasticity. The first term is given in the data, and the second is estimated using control functions (Olley and Pakes 1996).

For TFP, I use the utilization adjusted TFP series from Fernald. The series is quarterly, and I construct the yearly counterpart to match the markups data by taking the mean across 4 quarters.

2.2 Empirical strategy and results

Consider an economy with labour as the only factor of production: $y_t = A_t l_t$, where y_t, A_t, l_t are respectively output, TFP, and labour. Households have preferences over consumption and labour $\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\psi}}{1+\psi}\right]$, and let p_t, w_t be the price of c_t and l_t respectively. The consumption - labour Euler equation implies $l_t^{\psi} c_t^{\sigma} = w_t/p_t$. Define aggregate markup as $\mu_t = p_t / (w_t/A_t)$. Imposing $c_t = y_t$, we have

$$\left[\frac{y_t}{A_t}\right]^{\psi} y_t^{\sigma} = \frac{A_t}{\mu_t} \Rightarrow y_t = A_t^{\frac{1+\psi}{\sigma+\psi}} \mu_t^{-\frac{1}{\sigma+\psi}}$$

Taking log deviations give $\hat{y}_t = \frac{1+\psi}{\sigma+\psi}\hat{A}_t - \frac{1}{\sigma+\psi}\hat{\mu}_t$, where hat variables denote log deviations from the variable's mean.

The main thing to note is how markups affect output with the opposite sign as TFP. Intuitively, markups impose a wedge between the marginal rate of substitution between labour and consumption (labour demand), and the marginal product of labour (labour supply). Higher markups reduce labour and decrease output, the opposite effect of an increase in TFP. If a rise in TFP coincides with an increase in markups, then the effect of TFP on output would be dampen. This implies a decrease in fluctuations in output associated with TFP. The mechanism above generalizes to the case of decreasing returns to scale, as well as models with multiple factors of production.

To see whether the comovement between aggregate markups and TFP has changed, I run the regression of $\hat{\mu}_t$ on $\hat{A}_t$ with an indicator for after 1985:

$$\hat{\mu}_t = \alpha_1 \hat{A}_t + \alpha_2 \hat{A}_t \times 1_{\{t \geq 1985\}} + \epsilon_t$$

Aggregate markup for each year is the sales weighted average of individual firms' markups. Both markups and TFP are detrended using a HP filter with a smoothing parameter of 6.25 for yearly data.

The results of the above regression are given in Table 1. There is a significant increase in the comovement of markups with TFP post 1985. Compared to the 1955-1984 period, a 1% increase in TFP in 1985-2016 is associated with an increase in markups that is 0.67% points higher.

The link from concentration to aggregate markups comovements is as follows. Empirically, larger firms pass through less of their cost shocks onto prices. Following a positive TFP shock, marginal costs fall. Since markups is price on marginal costs, imperfect pass through implies that price do not decrease one for one with marginal costs, hence markups also increase. With an increase in concentration, aggregate markups take on more characteristics of large firms' markups. Higher concentration also means these firms produce higher fractions of output within their markets, which affects their cost pass through.

I first run the regression of firm markups on their own productivity fluctuations, to verify that larger firms have markups that covary more strongly with their own productivity. I include an indicator for large firms and indicators for time periods, to see the relationship pre and post 1985. The regression is

$$\begin{aligned}\hat{\mu}_{it} &= \beta_1 \hat{A}_{it} \times 1_{\{t<1985\}} + \beta_2 \hat{A}_{it} \times 1_{\{i_t \in large_t\}} \times 1_{\{t<1985\}} \\ &\quad + \beta_3 \hat{A}_{it} \times 1_{\{t\geq 1985\}} + \beta_4 \hat{A}_{it} \times 1_{\{i_t \in large_t\}} \times 1_{\{t\geq 1985\}} + \epsilon_{it}\end{aligned}$$

The results are given in Table 2. TFP is constructed in the method of Olley and Pakes (2016), following Imrohoroglu and Tuzel (2014), with more details in the appendix. Ideally log markups and log TFP should be detrended at the firm level, but there are missing data for firms across years for this specification. Instead I add in firm and time fixed effects. A firm is large in a given year if it has sales above 90th quantile of its 4 digit NAICS sales for that year. Estimates of β_2 and β_4 are positive, which give evidence that larger firms have markups that covary more strongly with their own productivity.

The question of interest is how firm markups comove with aggregate TFP. I run the regression of $\hat{\mu}_{it}$ on $\hat{A}_t$, with an indicator for large firms and indicators for post 1985:

$$\begin{aligned}\hat{\mu}_{it} &= \beta_1 \hat{A}_{t} \times 1_{\{t<1985\}} + \beta_2 \hat{A}_{t} \times 1_{\{i_t \in large_t\}} \times 1_{\{t<1985\}} \\ &\quad + \beta_3 \hat{A}_{t} \times 1_{\{t\geq 1985\}} + \beta_4 \hat{A}_{t} \times 1_{\{i_t \in large_t\}} \times 1_{\{t\geq 1985\}} + \epsilon_{it}\end{aligned}$$

The results are given in Table 3. Here log TFP is detrended using the HP filter. I consider two specifications: (1) log markups individually detrended[1], and (2) log markups with firm fixed effects. For specification (1), the estimates for β_2 and β_4 are positive, albeit with large standard errors. These estimates suggest that markups for large firms covary more with aggregate TFP compared to small firms, and this hold for both the pre and post 1985 periods. Specification (2) has β_2 slightly negative. Note that the comovements between

[1] There are less missing data if we do not construct firm level TFP

individual markups and aggregate TFP have risen post 1985, for both large and small firms. Since the estimates are of publicly traded firms, firms not classified as large by my definition may still produce a sizable fraction of output within its relevant market. It may be that these firms have increased their market share in their respective markets following the increase in concentration, leading them to have lower pass through hence higher markups comovements.

To assess how much concentration contributes to the change in aggregate markups comovements with TFP, and how much it contributes to the decrease in volatility, I now turn to the model with endogenous markup determination.

3 Model

Time is discrete and agents are infinitely lived. A representative final good producer aggregates intermediate goods into consumption bundles for households. There is a continuum of sectors of unit mass indexed by j producing intermediate goods. In each sector, there is a pair of positive mass firms indexed by $i \in \{1, 2\}$, which I also refer to as either large firms or duopolists. There is also a continuum of zero mass firms indexed by k, which I also refer to as small firms. Denote aggregate states by S and sectoral states by s, where the components of these states are detailed later in the paper. In the model, I normalize all prices by the wage.

3.1 Households

Households supply labour, demand consumption, have access to set of state contingent short term bonds, and own firms. Their recursive problem can be set up as

$$\mathbf{W}(S, B) = \max_{c_i(s), N, B'(S')} \log C - L + \beta E\left[\mathbf{W}(S', B(S'))\right] \quad s.t.$$

$$P(S)C(S) + \int Q(S, S')B'(S')dS' \leq L + B(S) + \Pi(S)$$

The utility specification here assumes that the intertemporal elasticity of substitution is 1, and that the inverse of the Frisch elasticity of labour is 0. The linear labour disutility gives rise to a constant stochastic discount factor: From the Euler equations,

$$1 = P(S)C(S), \quad Q(S, S') = \beta \frac{P(S)C(S)}{P(S')C(S')}$$

which implies $Q(S, S') = \beta$.

3.2 Final goods producer

A representative final goods producers aggregates intermediate goods into consumption bundles and sells them to the household. They solve the static problem of

$$\max PC - \int \left[p_{1,j}c_{1,j} + p_{2,j}c_{2,j} + \int p_{k,j}c_{k,j}dk \right] dj, \quad \text{where}$$

$$C = \exp\left[\int \log c_j dj \right]$$

$$c_j = \left[\left(\frac{M}{2}\right)^{\frac{1}{\eta}} \left[(z_{1,j}c_{1,j})^{\frac{\eta-1}{\eta}} + (z_{2,j}c_{2,j})^{\frac{\eta-1}{\eta}} \right] + (1-M)^{\frac{1}{\eta}} \int (z_{k,j}c_{k,j})^{\frac{\eta-1}{\eta}} dk \right]^{\frac{\eta}{\eta-1}}$$

The production function for aggregating intermediate goods is nested CES a la Atkeson and Burstein (2008). In the inner nest, intermediate goods enter with a quality shock z_i that evolves according to a random walk

$$\log z'_{ij} = \log z_{ij} + \sigma_z \epsilon'_{ij}$$

There is also a production weight M, that varies the importance of duopolist goods vs small firm goods in production. Increasing M will generate an increase in concentration.

I remain agnostic about the cause of the increase in concentration. Taken as face value, varying M here corresponds to changing the production technology. However, the setup is more general than just this. In the appendix, I show that the result of this setup is akin to different setups where the increase in concentration is from large firms growing more productive than small firms, or where the mass of small firms decreases due to, say, entry barriers.

Demand for intermediate inputs is given by

$$c_{i,j} = z_{i,j}^{\eta-1} \frac{M}{2} \left(\frac{p_{i,j}}{p_j}\right)^{-\eta} \left(\frac{p_j}{P}\right)^{-1} C$$

$$c_{k,j} = z_{i,k}^{\eta-1} (1-M) \left(\frac{p_{k,j}}{p_j}\right)^{-\eta} \left(\frac{p_j}{P}\right)^{-1} C, \quad \text{where}$$

$$p_j = \left[\frac{M}{2} \left[\left(\frac{p_{1,j}}{z_{1,j}}\right)^{1-\eta} + \left(\frac{p_{2,j}}{z_{2,j}}\right)^{1-\eta} \right] + (1-M) \int p_{k,j}^{1-\eta} dk \right]^{\frac{1}{1-\eta}}$$

In equilibrium, the price of the final good has to satisfy

$$P = \exp\left[\int \log p_j dj\right]$$

3.3 Intermediate goods firms

Firms are owned by households hence discount future payoffs at rate $Q(S, S') = \beta$. Marginal costs are given by z_{ij}/A^{ϕ} for large firms and z_{kj}/A for small firms. I make two assumptions in this specification of marginal costs. First is that the quality shocks also enter into marginal costs: A firm producing a higher quality good, which raises its demand, also has to expend more resources. This assumption follows Midrigan (2011), and is used generate a dispersion in prices and markups while reducing the state space and maintaining computational tractability, as I show below.

Second is that the aggregate cost shocks A affect the marginal cost of large and small firms differently. Specifically, I assume that $\phi > 1$ so that large firms are more affected by aggregate shocks. This could be interpreted as large firms are more integrated with the whole economy, hence more exposed to aggregtate fluctuations. I assume that A evolves according to

$$\log A' = \rho_A \log A + \sigma_A \epsilon'_A$$

The aggregate state S consists of the aggregate cost shock A along with the distribution of sectoral states. I show below that, under the assumption of log consumption and log outer CES nest, the distribution of sectoral states is redundant. The sectoral state s is the distribution of previous prices and quality shocks for the zero mass firms (denoted by λ_{kj}), along with the previous price and quality shocks of the positive mass firms. The timing is as follows: After firms observe the sectoral state, they draw independent menu costs ζ_{ij} that are private information for the firms. They then simultaneously choose whether to adjust prices, and their new prices if they adjust.

I assume that the distribution of menu costs is $H^l(\zeta) = U[0, \frac{M}{2}\bar{\zeta}]$ for large firms and $H^s(\zeta)U[0, (1-M)\bar{\zeta}]$ small firms. The scaling by $M/2$ and $(1-M)$ is so that firms expend a roughly constant share of revenue on price adjustment (around 1.3%, cf Klenow and Willis 2016), even as concentration varies.

3.3.1 Large firms

Here I drop sector subsripts j. Profits exclusive of adjustment are given by $\pi_i = c_i(p_i, p_{-i}, s, S)\left(p_i - z_i/A^{\phi}\right)$, where $c_i(p_i, p_{-i}, s, S)$ is the final goods producer demand. The firm's recursive problem is

$$V_i(s, S, \zeta_i) = \max_{\phi_i \in \{0,1\}} \phi_i \left[V_i^{adj}(s, S) - \zeta_i\right] + (1-\phi_i) V_i^{stay}(s, S)$$

$$\begin{aligned} V_i^{adj}(s, S) = \max_{p_i^*} \int & \Big[\overbrace{\phi_{-i}(s, S, \zeta_{-i}) \left\{\pi_i(p_i^*, p_{-i}^*(s, S), s, S) + E\left[\beta V_i(s'_{adj}, S', \zeta_i')\right]\right\}}^{\text{Rival adjusts}} \\ & + \underbrace{(1-\phi_{-i}(s, S, \zeta_{-i})) \left\{\pi_i(p_i^*, p_{-i}, s, S) + E\left[\beta V_i(s'_{adj}, S', \zeta_i')\right]\right\}}_{\text{Rival does not adjust}} \Big] dH^l(\zeta_{-i}) \end{aligned}$$

$$\begin{aligned} V_i^{stay}(s, S) = \int & \Big[\phi_{-i}(s, S, \zeta_{-i}) \left\{\pi_i(p_i, p_{-i}^*(s, S), s, S) + E\left[\beta V_i(s'_{stay}, S', \zeta_i')\right]\right\} \\ & + (1-\phi_{-i}(s, S, \zeta_{-i})) \left\{\pi_i(p_i, p_{-i}, s, S) + E\left[\beta V_i(s'_{stay}, S', \zeta_i')\right]\right\} \Big] d^l H(\zeta_{-i}) \end{aligned}$$

$$\begin{aligned} s'_{adj} = & \phi_{-i}(s, S, \zeta_{-i}) \times \left(p_i^*, p_{-i}^*(s, S), z_i', z_{-i}', \lambda_k'\right) \\ & + (1-\phi_{-i}(s, S, \zeta_{-i})) \times \left(p_i^*, p_{-i}, z_i', z_{-i}', \lambda_k'\right) \end{aligned}$$

$$\begin{aligned} s'_{stay} = & \phi_{-i}(s, S, \zeta_{-i}) \times \left(p_i, p_{-i}^*(s, S), z_i', z_{-i}', \lambda_k'\right) \\ & + (1-\phi_{-i}(s, S, \zeta_{-i})) \times \left(p_i, p_{-i}, z_i', z_{-i}', \lambda_k'\right) \end{aligned}$$

Above, ϕ denote policy for adjustment, which takes value $\{0, 1\}$. Firm i, after drawing their adjustment cost, decides whether to pay the adjustment cost and gain adjustment value V_i^{adj}, or keep their current price and gain staying value V_i^{stay}. In V_i^{adj}, the firm chooses the price that maximizes current profits plus the discounted expected value under the new price and tomorrow's states. In V_i^{stay}, the firm carries the previous price through, getting the profit under that price, plus the discounted expected value under that price and tomorrow's states. The firm has to take into account whether its positive mass rival adjusts as well as their optimal price conditional on adjustment. It integrates over the distribution of its rival unobserved menu costs, taking the rival's adjustment and pricing policies ϕ_{-i}, p_{-i}^* as given. I restrict to only Markov Perfect Equilibriums.

As menu costs are sunk, p^*_{-i} is independent of ζ_{-i}. Along with ζ_{-i} being an independent draw, it is sufficient to know only the probability of price changes $\gamma_{-i}(s,S) = \int \phi_{-i}(s,S,\zeta_{-i})dH^l(\zeta_{-i})$. ζ_i is an independent draw as well, and I can integrate it out of the continuation value and redefine V_i:

$$V_i(s,S) = \int \max\left\{V_i^{adj}(s,S) - \zeta_i, V_i^{stay}(s,S)\right\} dH(\zeta_i)$$

$$\begin{aligned}V_i^{adj}(s,S) = \max_{p_i^*} & \overbrace{\gamma_{-i}(s,S)\left\{\pi_i(p_i^*, p_{-i}^*(s,S), s, S) + E\left[\beta V_i(s'_{adj}, S')\right]\right\}}^{\text{Rival adjusts}} \\ & + \underbrace{(1-\gamma_{-i}(s,S))\left\{\pi_i(p_i^*, p_{-i}, s, S) + E\left[\beta V_i(s'_{adj}, S')\right]\right\}}_{\text{Rival does not adjust}}\end{aligned}$$

$$\begin{aligned}V_i^{stay}(s,S) = & \gamma_{-i}(s,S)\left\{\pi_i(p_i, p_{-i}^*(s,S), s, S) + E\left[\beta V_i(s'_{stay}, S')\right]\right\} \\ & + (1-\gamma_{-i}(s,S))\left\{\pi_i(p_i, p_{-i}, s, S) + E\left[\beta V_i(s'_{stay}, S')\right]\right\}\end{aligned}$$

3.3.2 Small firms

Profits exclusive of adjustment are given by $\pi_k(p_k, z_k, s, S) = c_k(p_k, z_k, s, S)(p_k - z_k/A)$. For small firms, they have to consider both of the positive mass firms' adjustment and pricing policies. Similar to above, due to the menu costs being sunk and independent, I can integrate them out and write the recursive problem as

$$V_k(p_k, s, S) = \int \max\left\{V_k^{adj}(p_k, s, S) - \zeta_k, V_k^{stay}(p_k, s, S)\right\} dH(\zeta_k)$$

$$\begin{aligned}V_k^{adj}(p_k, s, S) = \max_{p_k^*} & \overbrace{\gamma_1(s,S)\Bigg[\gamma_2(s,S)\left\{\pi_k(p_k^*, p_1^*, p_2^*(s,S), s, S) + E\left[\beta V_k(p_k^*, s'_{adj}, S')\right]\right\}}^{\text{Both firm adjust}} \\ & + \overbrace{(1-\gamma_2(s,S))\left\{\pi_k(p_k^*, p_1^*, p_2, s, S) + E\left[\beta V_k(p_k^*, s'_{adj}, S')\right]\right\}}^{\text{Only firm 1 adjust}} \\ & + \underbrace{(1-\gamma_1(s,S))\Bigg[\gamma_2(s,S)\left\{\pi_k(p_k^*, p_1, p_2^*(s,S), s, S) + E\left[\beta V_k(p_k^*, s'_{stay}, S')\right]\right\}}_{\text{Only firm 2 adjust}} \\ & + \underbrace{(1-\gamma_2(s,S))\left\{\pi_k(p_k^*, p_1, p_2, s, S) + E\left[\beta V_k(p_k^*, s'_{stay}, S')\right]\right\}}_{\text{No firm adjust}}\Bigg]\end{aligned}$$

with a similar expression for V_k^{stay}.

3.4 Change of variables

Recall that the sectoral state consists of firms' distribution of both the quality shock and previous prices, both of which are continuous variables. Below, I leverage the assumption on the quality shock shifting both demand and costs in order to reduce one set of continuous state variables. Denote firm level markups by $\mu_{ij} = \frac{p_{ij}}{z_{ij}/A^{\phi}}$ for large firms and $\mu_{kj} = \frac{p_{kj}}{z_{kj}/A}$ for small firms. For sectors, define $\mu_j = Ap_j$. For the aggregate economy, define $\mu = AP$. From the definition of the sectoral price index p_j, we obtain

$$\mu_j = \left[\frac{M}{2}A^{(\phi-1)(\eta-1)}\left((\mu_{1j})^{1-\eta} + (\mu_{2j})^{1-\eta}\right) + (1-M)\int (\mu_{kj})^{1-\eta}\, dk\right]^{\frac{1}{1-\eta}}$$

From the equilibrium condition on the aggregate price index, we have

$$\mu = \exp\left[\int \log \mu_j dj\right] \tag{1}$$

In terms of these μs and using that $PC = 1$, profits can be written as

$$\tilde{\pi}_{ij} = \underbrace{\frac{M}{2}A^{(\phi-1)(\eta-1)}\left(\mu_{ij}\right)^{-\eta}\left(\mu_j\right)^{\eta-1}}_{\text{demand}}(\overbrace{\mu_{ij}}^{\text{price}} - \overbrace{1}^{\text{mc}}) \tag{2}$$

$$\tilde{\pi}_{kj} = (1-M)\left(\mu_{kj}\right)^{-\eta}\left(\mu_j\right)^{\eta-1}\left(\mu_{kj}-1\right) \tag{3}$$

The continuation state also needs to be re-expressed. Take the case of a large firm, and say it sets μ_{ij} this period. The corresponding state next period is the markup as if prices have not changed, ie $\mu'_{ij} = \frac{p_{ij}}{z'_{ij}/A'^{\phi}}$. Subbing in $\mu_{ij} = \frac{p_{ij}}{z_{ij}/A^{\phi}}$ and invoking the law of motion for z_{ij} give

$$\mu'_{ij} = \frac{\mu_{ij}}{\exp(\sigma_z \epsilon'_{ij})}\left(\frac{A'}{A}\right)^{\phi}$$

With this change of variables, the sectoral state now consists only of the distribution of small firms previous markups, and the large firms previous markups. The profits function in (2) takes a similar form of quantity times price minus (constant) marginal costs. Marginal costs have been normalized to 1, so that the price is effectively the markup that the firm sets.

The recursive problem for a large firm can now be written as

$$V_i(X) = \int \max \left\{ \underbrace{V_i^{adj}(X) - \zeta_i}_{\text{adjust prices}}, \underbrace{V_i^{stay}(X)}_{\text{keep prices}} \right\} dH_i(\zeta_i)$$

$$V_i^{adj}(X) = \max_{\mu_i^*} \overbrace{\gamma_{-i}(X)}^{\text{pr. rival adjust}} \left\{ \tilde{\pi}_i(\mu_i^*, \mu_{-i}^*(X), X) + \beta E \left[V_i\left(\frac{\mu_i^* A'^{\phi}}{\exp(\sigma_z \epsilon_i') A^{\phi}}, \frac{\mu_{-i}^*(X) A'^{\phi}}{\exp(\sigma_z \epsilon_{-i}') A^{\phi}}, A', s' \right) \right] \right\}$$
$$+ \underbrace{(1 - \gamma_{-i}(X))}_{\text{pr. rival keep price}} \left\{ \tilde{\pi}_i(\mu_i^*, \mu_{-i}, X) + \beta E \left[V_i\left(\frac{\mu_i^* A'^{\phi}}{\exp(\sigma_z \epsilon_i') A^{\phi}}, \frac{\mu_{-i} A'^{\phi}}{\exp(\sigma_z \epsilon_{-i}') A^{\phi}}, A', s' \right) \right] \right\}$$

where $X = (s, S)$. The value of staying and the problem for small firms is similar, which I leave out for brevity.

3.5 Equilibrium

A recursive equilibrium for the above economy consists of

1. Household value function $\mathbf{W}(S, B)$, consumption demand $C(S)$, labour supply $L(S)$, and bond demand $B'(S', S)$
2. Pricing functions $\mu(S)$, $Q(S, S')$
3. Final goods producer's intermediate goods demand $c_{ij}(s, S), c_{kj}(s, S)$
4. Value function $V_i(s, S)$ and policy $\mu_i^*(s, S), \gamma_i(s, S)$ for large firms
5. Value function $V_k(\mu_k, s, S)$ and policy $\mu_k^*(\mu_k, s, S), \gamma_k(\mu_k, s, S)$ for small firms
6. Law of motion $\Gamma_k(\lambda_k, s, S)$ for the distribution of small firm markups

such that

- Given pricing functions , the household value function satisfies the household's Bellman equation at $B = 0$; and household consumption, labour supply and bond demand is consistent with optimality conditions
- The pricing functions clear the bonds market: $B = 0$; and clears the labour market
- Aggregate price μ equals the index in (1) under the policies $\mu_k^*, \gamma_k, \mu_i^*, \gamma_i$

- Given goods demand and positive mass rival's policies, the large firm's value function solves the large firm's Bellman equation; and the policy functions are consistent with optimization

- Given goods demand and the positive mass firms' policies, the small firm value function solves the small firm's Bellman equation; and the policy functions are consistent with optimization

- The law of motion for λ_k is consistent with small firm policy: Let $X_k \subset R_+$ and the corresponding Borel algebra of X_k be $\mathcal{X}_k$. Then $\lambda_k : \mathcal{X}_k \to [0,1]$ and satisfies the law of motion, for all sets $K \in \mathcal{X}_k$ and states (μ_1, μ_2, A):

$$\lambda_k'(K) = \int_{X_k} \left\{ \gamma_k(\mu_k, \mu_1, \mu_2, A) 1_{\{\mu_k^*(\mu_k, \mu_1, \mu_2, A) \in K\}} + (1 - \gamma_k(\mu_k, \mu_1, \mu_2, A)) 1_{\{\mu_k \in K\}} \right\} d\lambda_k(\mu_1, \mu_2, A)$$

3.6 Approximation

The sectoral state consists of an infinite dimensional distribution, which makes solving impractical. Following Krussel and Smith (1998), I assume that firms approximate the relevant outcomes of the distribution with finite moments and other state variables. The distribution of small firm μ_{kj} goes into calculating $\int (\mu_{kj})^{1-\eta} dk$ and hence μ_j. Let $\mu_{Kj} = \left(\int (\mu_k)^{1-\eta} dk\right)^{\frac{1}{1-\eta}}$ and approximate it using

$$\hat{\mu}_{Kj}(\mu_{1j}, \mu_{2j}, \hat{\mu}_{Kj,-1}, A) = \exp(\beta_1 + \beta_2 \log \mu_{1j} + \beta_3 \log \mu_{2j} + \beta_4 \log \hat{\mu}_{Kj,-1} + \beta_5 \log A)$$

Note that when going into the next period, each small firm's markup will be scaled by A'/A, hence so will the distribution. In practice, I scale lagged $\hat{\mu}_{Kj}$ by A'/A when running the updating regression.

3.7 Mechanism

The model generates a difference between the responses of large and small firms to aggregate TFP shocks, through two channels. The first I refer to as the static channel, which operates through the differing effect of the aggregate shock on large and small firms' marginal cost. The second I refer to as the dynamic channel, which works through the interplay of strategic complementarities and adjustment frictions.

3.7.1 Static channel

Consider the model without adjustment costs. Firms would reset their price every period, and set markups according to the elasticity rule $\frac{\varepsilon_{ij}}{\varepsilon_{ij}-1}$, where ε_{ij} is the elasticity of demand that the firm faces. From the CES specification, zero mass firms have constant elasticity η, so that their markups are $\frac{\eta}{\eta-1}$. For duopolists,

$$\varepsilon_{ij} = s_{ij} + \eta(1 - s_{ij})$$

Here, the elasticity they face is a weighted sum of the outer elasticity (1) and inner elasticity (η). The weight is their revenue share within the sector, $\frac{p_{ij}c_{ij}}{p_j c_j}$, which is

$$s_{ij} = \frac{\frac{M}{2}A^{(\phi-1)(\eta-1)}\left(\mu_{ij}\right)^{1-\eta}}{\frac{M}{2}A^{(\phi-1)(\eta-1)}\left(\left(\mu_{ij}\right)^{1-\eta} + \left(\mu_{-ij}\right)^{1-\eta}\right) + (1-M)\left(\eta/(\eta-1)\right)^{1-\eta}}$$

The interpretation is that as the firm gets larger, it competes more with firms in other sectors as opposed to firms within its sector, hence take on the (lower) elasticity of the outer nest.

Following an increase in A, for $\phi > 1$, s_{ij} would increase if μ_{ij} and μ_{-ij} remains constant. This generates a decrease in the elasticity ε_{ij}, increasing large firms markups. The equilibrium is that both μ_{ij} and μ_{-ij} increase, prompting an increase in μ_j and μ. This counteracts the effect of the increase in A on output.

The effect of this channel varies with M, but in a non-monotonic manner. For $M = 0$ or $M = 1$, a change in A does not change s_{ij}. The effect above is present for $M \in (0, 1)$. This is consistent with an U shaped relationship between cost pass through and firm size (Auer and Schoenle 2016, Amiti et. al. 2019).

3.7.2 Dynamic channel

For intuition, I consider a simplified static markup adjustment game. There are two players/firms $i \in \{1, 2\}$. Payoffs exclusive of adjustment costs are given by the second order Taylor approximation of the following profit function around the Bertrand Nash Equilibrium $\bar{\mu}$:

$$\pi_{ij}(\mu_{ij}, \mu_{-ij}) = \frac{M}{2}\mu_{ij}^{-\eta}\mu_j^{\eta-1}(\mu_{ij} - 1), \quad \text{where}$$

$$\mu_j = \left[\frac{M}{2}\left(\mu_{ij}^{1-\eta} + \mu_{-ij}^{1-\eta}\right) + (1-M)\left(\frac{\eta}{\eta-1}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

The function above is the same as equation (2), except for that I assume $\int \left(\mu_{kj}\right)^{1-\eta} dk = \left(\frac{\eta}{\eta-1}\right)^{1-\eta}$.

The two firms start with initial markups $(\bar{\mu}(1+\epsilon), \bar{\mu}(1+\epsilon))$, $\epsilon > 0$. The interpretation is that TFP has increased, leading to a decrease in marginal costs which would increase markups if firms do not adjust. They simultaneously choose whether or not to adjust μ_{ij}, facing fixed cost of adjustment $\frac{M}{2}\zeta$. For this game, I am mainly concerned with the region where firms do not adjust, ie where μ_{ij} increases along with A; and how that region changes as I vary M.

The second order Taylor approximation is

$$\hat{\pi}_i = \bar{\pi} + (\mu_i - \bar{\mu})\,A + (\mu_{-i} - \bar{\mu})\,B + \frac{(\mu_i - \bar{\mu})^2}{2}C + \frac{(\mu_{-i} - \bar{\mu})^2}{2}D + (\mu_i - \bar{\mu})\,(\mu_{-i} - \bar{\mu})\,E$$

where notable $C = \frac{\partial^2 \pi_{ij}(\bar{\mu},\bar{\mu})}{\partial \mu_{ij}^2}$ is the loss from deviating from the optimal $\bar{\mu}$ and $E = \frac{\partial^2 \pi_{ij}(\bar{\mu},\bar{\mu})}{\partial \mu_{ij}\partial \mu_{-ij}}$ is the strategic term. One can show that $C < 0$, the second order condition for $\bar{\mu}$ to be optimal, and that $E > 0$, ie the profit function exhibits strategic complementarities in markups.

If the firm adjust, given rival's markup μ_{-ij}, first order conditions imply that they adjust to $\mu_{ij}^*(\mu_{-ij}) = \bar{\mu} + \frac{(\mu_{-ij} - \bar{\mu})E}{-C}$. As $E > 0$ and $C < 0$, the optimal reset markup is increasing in the rival's markup.

The no adjustment equilibrium can be sustained when

$$\hat{\pi}_{ij}\left(\bar{\mu}(1+\epsilon), \bar{\mu}(1+\epsilon)\right) \geq \hat{\pi}_{ij}\left(\mu_{ij}^*\left(\bar{\mu}(1+\epsilon)\right), \bar{\mu}(1+\epsilon)\right) - \zeta$$

That is, given that the rival does not adjust, the profit from the firm keeping its markup is higher than the profit under reseting optimally and paying the fixed adjustment cost. The above condition simplifies to a condition on the size of the deviation from $\bar{\mu}$:

$$\epsilon \leq \frac{1}{\bar{\mu}}\sqrt{\frac{-2\zeta C}{(C+E)^2}} := \mu^u$$

which gives an inaction region in which no adjustment can be sustained. In the appendix, I show that (1) $|C|$ is decreasing in M, (2) E is in increasing in M, and (3) the bound μ^u is increasing in M. The intuition is that as the firms get larger: (1) their elasticity of demand falls, which decreases the loss from deviating from $\bar{\mu}$; and (2) their rival gets larger hence impact their decisions more, increasing the degree of strategic complementary. The combination of these effects widens the inaction region, generating more rigidity in adjustment as firm concentration increase. As a result, markups covaries more with TFP.

4 Results

4.1 Parameters

The model is calibrated at the quarterly frequency. I set the discount rate to $\beta = 0.95^{1/4}$ and aggregate shock persistence to $\rho_A = 0.8$. The baseline has the weighing parameter M set to target concentration of 0.2[2]. The lower elasticity of substitution η is set to target an aggregate markup of 1.3 - DeLoecker et. al. (2020) estimates for the pre 1990s period. The (unscaled) upper bound $\bar{\zeta}$ of the uniform menu cost distribution and idiosyncratic shock std σ_z is set to match a mean price duration of 1 year, and mean magnitude of price change of 20%[3]. The relative exposure of large firms to TFP shocks, ϕ, is externally estimated: I estimate individual firm TFP using the method of Olley and Pakes (1996), following Imrohoroglu and Tuzel (2014), then regress log firm TFP on log aggregate TFP with an interaction on whether the firm is large. ϕ is then the ratio of the estimates for large firms vs small firms.

In the model, since large firms have higher exposure to TFP shocks, increasing concentration would mechanically increase volatility. Ideally, the contribution of the volatility of TFP to the volatility of output should be held constant. I set the std of aggregate shock innovation $\sigma_A = \frac{\hat{\sigma}_A}{M\phi+(1-M)}$, where $\hat{\sigma}_A$ is fixed and set to target an output std of 1.77% at the baseline M. This parameterization has that the contribution of the volatility of TFP to the volatility of output is the same at $M = 0$ and $M = 1$. To see how the parameterization does for $M \in (0, 1)$, I fix $\phi = 1$ and vary M. Note that at $\phi = 1$, the contribution of the volatility of TFP to the volatility of output is constant. I then calculate observed TFP - $\widehat{TFP}_t = C_t/L_t$,and calculate its log standard deviation. I do the same for my value $\phi > 1$, then compare how the log standard deviations change as I vary concentration. The changes are quite similar.

These parameters are summarized in table 4.

4.2 Results

To see how well the model does to match the differences in response to TFP across small and large firms, I simulate the economy across time and regress demeaned log aggregate markups on log TFP separately for small and large firms. I then take the difference between the

[2]Concentration is hard to measure, as it requires pinning down the relevant market that the firm competes in. I target 0.2 as it corresponds to the average of concentration across 4-digit industries in the 1980s. The results have that the relation between concentration and output volatility is quite linear, so that the effect of increasing concentration is robust to the initial level.

[3]Prices in the model are real. Here, my targets of mean duration and magnitude are from estimates of nominal prices for the post 1980 period. My assumption is that these two objects do not differ so much, given the low inflation since the 1980s.

estimate for small firms and that for large firms. I do this for the values of CR2 concentration - average share of sector output produced by the two large firms - at 0.2 and 0.3. The simulated results are compared with the data estimates for pre and post 1985, and is shown in Table 5. I take averages across 4 quarters before taking logs so that the simulated results are comparable to the yearly data estimates. Whereas the data estimates have decreased post 1985, the simulated results increase with concentration. My conjecture is that, since Compustat firms are likely to be large, the decline in the data estimates could be the result of the firms that I do not classify as large becoming larger and looking more similar to the firms that I do classify as large. If this conjecture is correct, then the actual differences between large and (actually) small firms in the data should be larger than the Compustat estimates. The model then underestimates the difference, and would likely understate the effect of increasing concentration on volatility.

Figure **1** plots the change in std of aggregate output as I increase concentration from the baseline of 0.2. Evidence from Autor et. al. (2020) suggests that concentration has risen by 10 percentage points post 1980s. Increasing concentration in the model decreases aggregate output std by 0.017 percentage point, which is roughly 2% of the 0.8 percentage point decrease in output volatility associated with the Great Moderation. As discussed above, this is possibly an understatement of the effect of rising concentration.

To see the importance of the static and the dynamic channels, I consider the change in std of aggregate output as concentration varies for the case of (1) no adjustment costs ($\bar{\zeta} = 0$) and (2) same exposures ($\phi = 1$). (1) captures only the static channel while (2) captures only the dynamic channel. Figure 3 plots these changes, along with the baseline. Most of the decrease in output volatility is being captured by the static channel. The dynamic channel matters more only for very high levels of concentration, which is not shown here.

5 Conclusion

In this paper I explored a link between rising concentration and decreasing output volatility. Empirically, aggregate markups covary more strongly with TFP in the period post 1985. Estimates suggest that large firms' markups covary more strongly with aggregate TFP compared to small firms', in the periods pre and post 1985. The rise in concentration during that period links the former to the latter. Large firms become larger, leading their markups to covary even more strongly with aggregate TFP. There is also a composition effect. With more weight on large firms, aggregate markups inherits more of large firm markups characteristics hence covary more strongly with aggregate TFP. Higher markups widens the wedge between input supply and demand, decreasing input and output quantities. This goes against the effects of higher TFP, which dampens the effect of TFP movements on output volatility.

To understand the mechanism behind different firms' markup response to TFP, and to quantify the effect of rising concentration, I built a model incorporating large positive mass firms and small zero mass firms. There are two channels in the model that generate variable markups. First is that large firms are more affected by TFP shocks compared to small firms. The second is the existence of adjustment frictions, whereby the large firms' strategic complementarities in pricing amplifies the rigidity caused by these frictions. In the model, a 10 percentage point increase in the share of products produced by large firms decreases aggregate output std by 0.017 percentage point. However, the model potentially underestimates the effect of increasing concentration on output volatility.

A potential way to increase the effect would be to consider input output linkages and incorporate network effects. For future works, it would also be fruitful to introduce more shocks into the model, as TFP is not the only driving factor of output volatility.

References

Ä°mrohoroÄlu, AyÅe and Åelale TÃŒzel (Aug. 1, 2014). "Firm-Level Productivity, Risk, and Return". In: *Management Science* 60.8. Publisher: INFORMS, pp. 2073–2090.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings (Nov. 1, 2019). "International Shocks, Variable Markups, and Domestic Prices". In: *The Review of Economic Studies* 86.6, pp. 2356–2402.

Atkeson, Andrew and Ariel Burstein (Dec. 2008). "Pricing-to-Market, Trade Costs, and International Relative Prices". In: *American Economic Review* 98.5, pp. 1998–2031.

Auer, Raphael A. and Raphael S. Schoenle (Jan. 2016). "Market structure and exchange rate pass-through". In: *Journal of International Economics* 98, pp. 60–77.

Autor, David et al. (May 1, 2020). "The Fall of the Labor Share and the Rise of Superstar Firms*". In: *The Quarterly Journal of Economics* 135.2, pp. 645–709.

Burstein, Ariel and Gita Gopinath (Jan. 1, 2014). "Chapter 7 - International Prices and Exchange Rates". In: *Handbook of International Economics*. Ed. by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff. Vol. 4. Handbook of International Economics. Elsevier, pp. 391–451.

Caballero, Ricardo J. and Eduardo M. R. A. Engel (Sept. 1, 2007). "Price stickiness in Ss models: New interpretations of old results". In: *Journal of Monetary Economics*. Supplement issue: October 20-21 2006 Research Conference on 'Microeconomic Adjustment and Macroeconomic Dynamics' Sponsored by the Swiss National Bank (http://www.snb.ch) and Study Center Gerzensee (www.szgerzensee.ch) 54, pp. 100–121.

Carvalho, Vasco and Xavier Gabaix (Aug. 2013). "The Great Diversification and Its Undoing". In: *American Economic Review* 103.5, pp. 1697–1727.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger (May 1, 2020). "The Rise of Market Power and the Macroeconomic Implications*". In: *The Quarterly Journal of Economics* 135.2, pp. 561–644.

GalÃ, Jordi and Luca Gambetti (Jan. 1, 2009). "On the Sources of the Great Moderation". In: *American Economic Journal: Macroeconomics* 1.1, pp. 26–57.

Golosov, Mikhail and RobertÂ E. LucasÂ Jr. (2007). "Menu Costs and Phillips Curves". In: *Journal of Political Economy* 115.2. Publisher: The University of Chicago Press, pp. 171–199.

Klenow, Peter J. and Jonathan L. Willis (2016). "Real Rigidities and Nominal Price Changes". In: *Economica* 83.331. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/ecca.12191, pp. 443–472.

Krusell, Per and Jr. Smith AnthonyÂ A. (Oct. 1, 1998). “Income and Wealth Heterogeneity in the Macroeconomy”. In: *Journal of Political Economy* 106.5. Publisher: The University of Chicago Press, pp. 867–896.

Midrigan, Virgiliu (2011). “MENU COSTS, MULTIPRODUCT FIRMS, AND AGGREGATE FLUCTUATIONS”. In: *Econometrica* 79.4. Publisher: [Wiley, Econometric Society], pp. 1139–1180.

Mongey, Simon (Sept. 2021). *Market Structure and Monetary Non-neutrality*. Working Paper 29233. Series: Working Paper Series. National Bureau of Economic Research.

Olley, G. Steven and Ariel Pakes (1996). “The Dynamics of Productivity in the Telecommunications Equipment Industry”. In: *Econometrica* 64.6. Publisher: [Wiley, Econometric Society], pp. 1263–1297.

Stock, James H. and Mark W. Watson (Jan. 2003). “Has the Business Cycle Changed and Why?” In: *NBER Macroeconomics Annual 2002, Volume 17*. MIT Press, pp. 159–230.

Wang, Olivier and IvÃ¡n Werning (July 2020). *Dynamic Oligopoly and Price Stickiness*. Working Paper 27536. Series: Working Paper Series. National Bureau of Economic Research.

Figures and Tables

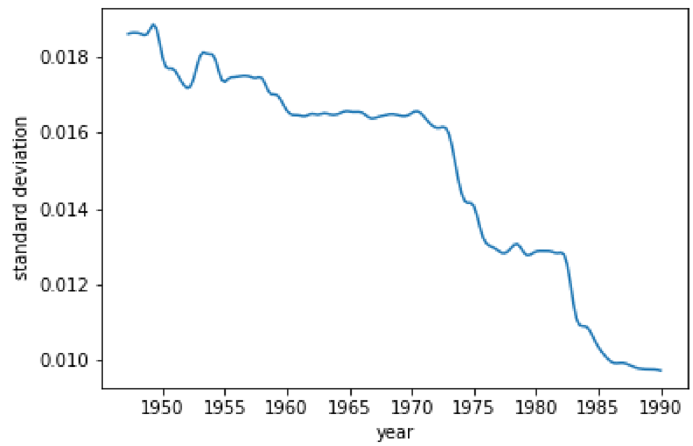

Figure 1: Rolling window of GDP standard deviation

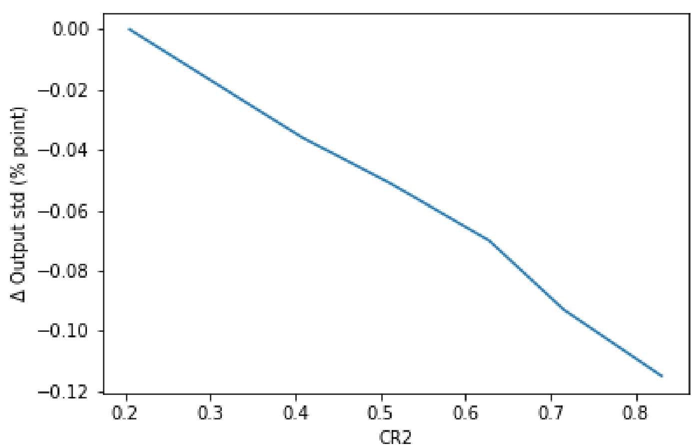

Figure 2: Output volatility over various concentration levels

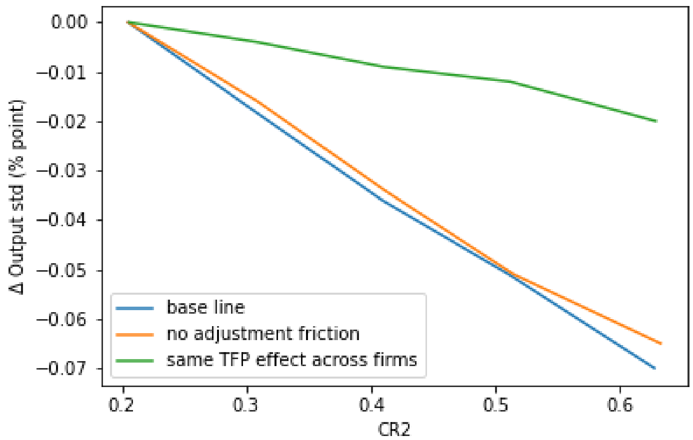

Figure 3: Static vs Dynamic channel contribution

Dependent variable: $\hat{\mu}_t$	
$\hat{A}_t$	-0.0679
	(0.207)
$\hat{A}_t \times 1_{\{t \geq 1985\}}$	0.6651
	(0.300)
N obs	62
Adjusted R^2	0.085

Table 1: Aggregate markups and TFP

Dependent variable	$\hat{\mu}_{it}$
$\hat{A}_{it} \times 1_{\{t<1985\}}$	0.1638
	(0.0025)
$\hat{A}_{it} \times 1_{\{i_t \in large_t\}} \times 1_{\{t<1985\}}$	0.0262
	(0.0036)
$\hat{A}_{it} \times 1_{\{t \geq 1985\}}$	0.1789
	(0.0016)
$\hat{A}_{it} \times 1_{\{i_t \in large_t\}} \times 1_{\{t \geq 1985\}}$	0.0403
	(0.0031)
Firm FE	Yes
Time FE	Yes
N obs	146751

Table 2: Firm level markups and firm level TFP

Dependent variable	$\hat{\mu}_{it}$	
	(1)	(2)
$\hat{A}_t \times 1_{\{t<1985\}}$	-0.2011	-0.3739
	(0.0744)	(0.1425)
$\hat{A}_t \times 1_{\{i_t \in large_t\}} \times 1_{\{t<1985\}}$	0.2235	0.4230
	(0.1754)	(0.3336)
$\hat{A}_t \times 1_{\{t\geq 1985\}}$	-0.0146	0.0894
	(0.0465)	(0.0896)
$\hat{A}_t \times 1_{\{i_t \in large_t\}} \times 1_{\{t\geq 1985\}}$	0.0770	-0.0337
	(0.1279)	(0.2447)
Firm FE	No	Yes
N obs	226211	226211

Table 3: Firm level markups and aggregate TFP

Parameter	Target	Value
β	Externally set	0.9873
ρ_A	Externally set	0.8
$\widehat{\sigma}_A$	1955-1985 output volatility	0.0135
η	1955-1990 markups	4.7
$\bar{\zeta}$	Mean price duration and	0.22
σ_z	mean magnitude of price change	0.105
ϕ	Large firm TFP response - External	1.25

Table 4: Calibration parameters

Concentration	$\beta_2^{duo} - \beta_2^{fringe}$	Data
0.205	0.039	0.2235
0.301	0.0623	0.077

Table 5: Firm level markups and TFP - simulated data

Appendix

Demand derivation for final goods producer

The profit maximization problem is

$$\max PC - \int \left[p_{1,j}c_{1,j} + p_{2,j}c_{2,j} + \int p_{k,j}c_{k,j}dk \right] dj, \quad \text{where}$$

$$C = \exp\left[\int \log c_j dj\right]$$

$$c_j = \left[\left(\frac{M}{2}\right)^{\frac{1}{\eta}} \left[(z_{1,j}c_{1,j})^{\frac{\eta-1}{\eta}} + (z_{2,j}c_{2,j})^{\frac{\eta-1}{\eta}} \right] + (1-M)^{\frac{1}{\eta}} \int (z_{k,j}c_{k,j})^{\frac{\eta-1}{\eta}} dk \right]^{\frac{\eta}{\eta-1}}$$

FOCs:

$$c_{1j} : \left(\frac{M}{2}\right)^{\frac{1}{\eta}} c_j^{\frac{1}{\eta}} z_{1j} (z_{1j}c_{1j})^{-\frac{1}{\eta}} C c_j^{-1} P = p_{1j}$$

$$c_{2j} : \left(\frac{M}{2}\right)^{\frac{1}{\eta}} c_j^{\frac{1}{\eta}} z_{2j} (z_{2j}c_{2j})^{-\frac{1}{\eta}} C c_j^{-1} P = p_{2j}$$

$$c_{kj} : (1-M)^{\frac{1}{\eta}} c_j^{\frac{1}{\eta}} z_{kj} (z_{kj}c_{kj})^{-\frac{1}{\eta}} C c_j^{-1} P = p_{kj}$$

$$c_{1j} = \left(\frac{M}{2}\right) z_{1j}^{-1} \left(P c_j^{\frac{1}{\eta}} \left(\frac{c_j}{C}\right)^{-1} z_{1j} p_{1j}^{-1} \right)^{\eta}$$

$$c_{2j} = \left(\frac{M}{2}\right) z_{2j}^{-1} \left(P c_j^{\frac{1}{\eta}} \left(\frac{c_j}{C}\right)^{-1} z_{2j} p_{2j}^{-1} \right)^{\eta}$$

$$c_{kj} = (1-M)\, z_{kj}^{-1} \left(P c_j^{\frac{1}{\eta}} \left(\frac{c_j}{C}\right)^{-1} z_{kj} p_{kj}^{-1} \right)^{\eta}$$

$$c_j = \left[\left(\frac{M}{2}\right) \left[\left(P c_j^{\frac{1}{\eta}} \left(\frac{c_j}{C}\right)^{-1} z_{1j} p_{1j}^{-1} \right)^{\eta-1} + \left(P c_j^{\frac{1}{\eta}} \left(\frac{c_j}{C}\right)^{-1} z_{2j} p_{2j}^{-1} \right)^{\eta-1} \right] + (1-M) \int \left(P c_j^{\frac{1}{\eta}} \left(\frac{c_j}{C}\right)^{-1} z_{kj} p_{kj}^{-1} \right)^{\eta-1} dk \right]$$

$$\left(P \left(\frac{c_j}{C}\right)^{-1} \right) = \left[\left(\frac{M}{2}\right) \left[\left(z_{1j} p_{1j}^{-1} \right)^{\eta-1} + \left(z_{2j} p_{2j}^{-1} \right)^{\eta-1} \right] + (1-M) \int \left(z_{kj} p_{kj}^{-1} \right)^{\eta-1} dk \right]^{\frac{1}{1-\eta}}$$

Define $p_j = \left[\left(\frac{M}{2}\right)\left[\left(\frac{p_{1j}}{z_{1j}}\right)^{1-\eta} + \left(\frac{p_{2j}}{z_{2j}}\right)^{1-\eta}\right] + (1-M)\int\left(\frac{p_{kj}}{z_{kj}}\right)^{1-\eta} dk\right]^{\frac{1}{1-\eta}}$. Then

$$c_{i,j} = z_{i,j}^{\eta-1}\frac{M}{2}\left(\frac{p_{i,j}}{p_j}\right)^{-\eta}\left(\frac{p_j}{P}\right)^{-1} C$$

$$c_{k,j} = z_{i,k}^{\eta-1}(1-M)\left(\frac{p_{k,j}}{p_j}\right)^{-\eta}\left(\frac{p_j}{P}\right)^{-1} C$$

Comparison to setup with changing productivity or changing small firm mass

Consider instead the model with final goods production

$$\max PC - \int\left[p_{1,j}c_{1,j} + p_{2,j}c_{2,j} + \int p_{k,j}c_{k,j}dk\right]dj, \quad \text{where}$$

$$C = \exp\left[\int \log c_j dj\right]$$

$$c_j = \left[\left[(z_{1,j}c_{1,j})^{\frac{\eta-1}{\eta}} + (z_{2,j}c_{2,j})^{\frac{\eta-1}{\eta}}\right] + \int^K (z_{k,j}c_{k,j})^{\frac{\eta-1}{\eta}} dk\right]^{\frac{\eta}{\eta-1}}$$

and intermediate firms' marginal costs given by $z_{ij}/\left(\gamma_h A^\phi\right)$ for duopolists and $z_{kj}/(\gamma_l A)$ for small firms. Here, demand for intermediate goods is given by

$$c_{i,j} = z_{i,j}^{\eta-1}\left(\frac{p_{i,j}}{p_j}\right)^{-\eta}\left(\frac{p_j}{P}\right)^{-1} C$$

$$c_{k,j} = z_{i,k}^{\eta-1}\left(\frac{p_{k,j}}{p_j}\right)^{-\eta}\left(\frac{p_j}{P}\right)^{-1} C, \quad \text{where}$$

$$p_j = \left[\left[\left(\frac{p_{1j}}{z_{1j}}\right)^{1-\eta} + \left(\frac{p_{2j}}{z_{2j}}\right)^{1-\eta}\right] + \int^K \left(\frac{p_{kj}}{z_{kj}}\right)^{1-\eta} dk\right]^{\frac{1}{1-\eta}}$$

Let $\mu_{ij} = \frac{p_{ij}}{z_{ij}/\left(\gamma_h A^\phi\right)}$ for large firms and $\mu_{kj} = \frac{p_{kj}}{z_{kj}/(\gamma_l A)}$ for small firms. Define $\mu_j = p_j A\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)^{\frac{1}{\eta-1}}$, so that

$$\begin{aligned}\mu_j &= \left[\left[\left(\frac{\mu_{1j}}{\gamma_h A^\phi}\right)^{1-\eta} + \left(\frac{\mu_{2j}}{\gamma_h A^\phi}\right)^{1-\eta}\right] + \int^K \left(\frac{\mu_{kj}}{\gamma_l A}\right)^{1-\eta} dk\right]^{\frac{1}{1-\eta}} A\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)^{\frac{1}{\eta-1}} \\ &= \left[\frac{\gamma_h^{\eta-1}}{\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)} A^{(\phi-1)(\eta-1)}\left[(\mu_{1j})^{1-\eta} + (\mu_{2j})^{1-\eta}\right] + \frac{\gamma_l^{\eta-1}}{\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)}\int^K (\mu_{kj})^{1-\eta} dk\right]^{\frac{1}{1-\eta}}\end{aligned}$$

Define $\mu = PA\left(2\gamma_h^{\eta-1} + \gamma_l^{\eta-1}\right)^{\frac{1}{\eta-1}}$. Profits are given by

$$\tilde{\pi}_{ij} = \frac{\gamma_h^{\eta-1}}{\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)} A^{(\phi-1)(\eta-1)} \left(\mu_{ij}\right)^{-\eta} \left(\mu_j\right)^{\eta-1} \left(\mu_{ij} - 1\right) \tag{4}$$

$$\tilde{\pi}_{kj} = \frac{\gamma_l^{\eta-1}}{\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)} \left(\mu_{kj}\right)^{-\eta} \left(\mu_j\right)^{\eta-1} \left(\mu_{kj} - 1\right) \tag{5}$$

Note that $C = \mu^{-1} A \left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)^{\frac{1}{\eta-1}}$, so that std of $\log C$ does not depend on the scaling constant $\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)^{\frac{1}{\eta-1}}$. Since zero mass firms are ex-ante identical,

$$\int^K \left(\mu_{kj}\right)^{1-\eta} dk = K \int \left(\mu_{kj}\right)^{1-\eta} dk$$

Then setting $\frac{M}{2} = \frac{\gamma_h^{\eta-1}}{\left(2\gamma_h^{\eta-1} + K\gamma_l^{\eta-1}\right)}$ will bring us back to the baseline model.

Static markup game

The second order Taylor approximation of

$$\pi_i = \mu_i^{-\eta} \mu_j^{\eta-1} (\mu_i - 1) \frac{M}{2}, \quad \text{where}$$

$$\mu_j = \left[\frac{M}{2}\left(\mu_i^{1-\eta} + \mu_{-i}^{1-\eta}\right) + (1-M)\left(\frac{\eta}{\eta-1}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

is (note $\partial\mu_j/\partial\mu_i = \mu_j^{\eta}\mu_i^{-\eta}\frac{M}{2}$)

$$\hat{\pi}_i = \bar{\pi} + (\mu_i - \bar{\mu}) A + (\mu_{-i} - \bar{\mu}) B + \frac{(\mu_i - \bar{\mu})^2}{2} C + \frac{(\mu_{-i} - \bar{\mu})^2}{2} D + (\mu_i - \bar{\mu})(\mu_{-i} - \bar{\mu}) E$$

$$
\begin{aligned}
A &= \bar{\mu}_j^{\eta-1}\left((1-\eta)\bar{\mu}^{-\eta}+\eta\bar{\mu}^{-\eta-1}\right)+\bar{\mu}^{-2\eta}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{2\eta-2}\frac{M}{2} \\
B &= \bar{\mu}^{-\eta}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-\eta}\frac{M}{2} \\
C &= \left((1-\eta)\bar{\mu}^{-\eta}+\eta\bar{\mu}^{-\eta-1}\right)(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-\eta}\frac{M}{2}+\bar{\mu}_j^{\eta-1}\left(-\eta(1-\eta)\bar{\mu}^{-\eta-1}-\eta(1+\eta)\bar{\mu}^{-\eta-2}\right) \\
&\quad+\frac{M}{2}(\eta-1)\left(\bar{\mu}^{-2\eta}(\bar{\mu}-1)(2\eta-2)\bar{\mu}_j^{2\eta-3}\bar{\mu}_j^{\eta}\bar{\mu}^{-\eta}\frac{M}{2}+\bar{\mu}_j^{2\eta-2}\left((1-2\eta)\bar{\mu}^{-2\eta}+2\eta\bar{\mu}^{-2\eta-1}\right)\right) \\
D &= \bar{\mu}^{-\eta}(\bar{\mu}-1)(\eta-1)\frac{M}{2}\left(-\eta\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-\eta-1}+\bar{\mu}^{-\eta}(2\eta-2)\bar{\mu}_j^{2\eta-3}\bar{\mu}_j^{\eta}\bar{\mu}^{-\eta}\frac{M}{2}\right) \\
E &= \bar{\mu}^{-\eta}\frac{M}{2}(\eta-1)\left(\bar{\mu}_j^{2\eta-2}\left((1-\eta)\bar{\mu}^{-\eta}+\eta\bar{\mu}^{-\eta-1}\right)+\bar{\mu}^{-\eta}(\bar{\mu}-1)(2\eta-2)\bar{\mu}_j^{2\eta-3}\bar{\mu}_j^{\eta}\bar{\mu}^{-\eta}\frac{M}{2}\right)
\end{aligned}
$$

Note that $A = 0$. Given that rival chooses μ_{-i}, if firm adjusts, they adjust to μ_i^*:

$$0 = (\mu_i^* - \bar{\mu})C + (\mu_{-i} - \bar{\mu})\,E$$

$$\mu_i^* = \bar{\mu} + \frac{(\mu_{-i} - \bar{\mu})\,E}{-C}$$

With players starting at (μ, μ), where $\mu = \bar{\mu}(1+\epsilon), \epsilon > 0$, there are 3 cases: Neither adjust, both adjust, only one adjust. The third case can be ruled out as $E > 0$ (to be shown below).

If the rival does not adjust, the firm will not adjust when

$$\bar{\pi} + (\epsilon\bar{\mu})\,B + \frac{(\epsilon\bar{\mu})^2}{2}C + \frac{(\epsilon\bar{\mu})^2}{2}D + (\epsilon\bar{\mu})^2\,E >$$

$$\bar{\pi} + (\epsilon\bar{\mu})\,B + \frac{\left(\frac{(\epsilon\bar{\mu})E}{-C}\right)^2}{2}C + \frac{(\epsilon\bar{\mu})^2}{2}D + \left(\frac{(\epsilon\bar{\mu})\,E}{-C}\right)(\epsilon\bar{\mu})\,E - \zeta$$

$$\Longleftrightarrow$$

$$\frac{(\epsilon\bar{\mu})^2}{2}C + (\epsilon\bar{\mu})^2\,E > -\frac{((\epsilon\bar{\mu})\,E)^2}{2C} - \zeta$$

$$\Longleftrightarrow$$

$$\epsilon < \frac{1}{\bar{\mu}}\sqrt{\frac{-2\zeta C}{(C+E)^2}}$$

If the rival adjusts, the firm adjusts when

$$\bar{\pi} - \zeta > \bar{\pi} + \frac{(\epsilon\bar{\mu})^2}{2}C$$

$$\iff$$

$$\epsilon > \frac{1}{\bar{\mu}}\sqrt{\frac{2\zeta}{-C}}$$

Note that $\sqrt{\frac{-2\zeta C}{(C+E)^2}} > \sqrt{\frac{2\zeta}{-C}}$ as $-C > E$. We have that $A = 0 \iff$

$$((1-\eta)\bar{\mu}+\eta) + \bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2} = 0$$

Using this,

$$\begin{aligned} C &= -\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}\left[((1-\eta)\bar{\mu}+\eta) + (\bar{\mu}-1)(\eta-1)\bar{\mu}^{-\eta+1}\bar{\mu}_j^{\eta-1}\frac{M}{2}\right] \\ &- \eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2} + \bar{\mu}(\eta-1)\bar{\mu}^{-2\eta-1}\bar{\mu}_j^{2\eta-2}\frac{M}{2} - \frac{M}{2}(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-2\eta} \\ &= -\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2} \end{aligned}$$

$$\begin{aligned} E &= \frac{M}{2}(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-2\eta-1}\left(-\left((1-\eta)\bar{\mu}+\eta\right)\right) \\ &= \left((1-\eta)\bar{\mu}+\eta\right)^2\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}(\bar{\mu}-1)^{-1} \end{aligned}$$

From here, we see $C < 0$ and $E > 0$. We have $\frac{dC}{d\bar{\mu}} > 0$, and

$$\begin{aligned} \frac{dE}{d\bar{\mu}} &= 2\left((1-\eta)\bar{\mu}+\eta\right)(1-\eta)\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}(\bar{\mu}-1)^{-1} \\ &+ \left((1-\eta)\bar{\mu}+\eta\right)^2(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-2\eta-2}(\bar{\mu}-1)^{-1} \\ &- \left((1-\eta)\bar{\mu}+\eta\right)^2\bar{\mu}_j^{\eta-1}(\eta+2)\bar{\mu}^{-\eta-3}(\bar{\mu}-1)^{-1} \\ &- \left((1-\eta)\bar{\mu}+\eta\right)^2\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}(\bar{\mu}-1)^{-2} \end{aligned}$$

This has the same sign as

$$\begin{aligned} (\bar{\mu}-1)\big(2(\eta-1)\bar{\mu}^{\eta} - ((1-\eta)\bar{\mu}+\eta)(\eta-1)\bar{\mu}_j^{\eta-1} + ((1-\eta)\bar{\mu}+\eta)(\eta+2)\bar{\mu}^{\eta-1}\big) + \bar{\mu}^{\eta}\left((1-\eta)\bar{\mu}+\eta\right) &= \\ \bar{\mu}^{\eta-1}\big((\eta-1)(\eta+1)\bar{\mu}^2 + \bar{\mu}2\eta(\eta+1) - \eta(\eta+2)\big) - \bar{\mu}_j^{\eta-1}(\eta-1)\big((\eta-1)\bar{\mu}^2 + \bar{\mu}(2\eta-1) - \eta\big) &\geq \\ \bar{\mu}^{\eta-1}\big((\eta-1)(\eta+1)\bar{\mu}^2 + \bar{\mu}2\eta(\eta+1) - \eta(\eta+2) - (\eta-1)\big((\eta-1)\bar{\mu}^2 + \bar{\mu}(2\eta-1) - \eta\big)\big) \end{aligned}$$

Since $((\eta-1)\bar{\mu}^2 + \bar{\mu}(2\eta-1) - \eta) > 0$ and $\bar{\mu} > \bar{\mu}_j$. This has the same sign as

$$\bar{\mu}^2 2(\eta-1) + \bar{\mu}\left(5\eta-1\right) - 3\eta$$

Now

$$\frac{\partial \bar{\mu}^2 2(\eta-1)+\bar{\mu}\,(5\eta-1)-3\eta}{\partial\bar{\mu}} = 4(\eta-1)\bar{\mu}+5\eta-1>0$$

and

$$\begin{aligned}\left(\bar{\mu}^2 2(\eta-1)+\bar{\mu}\,(5\eta-1)-3\eta\right)|_{\bar{\mu}=1} &= 2(\eta-1)+(5\eta-1)-3\eta \\ &= 2(\eta-1)+2\eta-1>0\end{aligned}$$

So

$$\frac{dE}{d\bar{\mu}}>0$$

We can write

$$\begin{aligned}E &= \frac{M}{2}(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-2\eta-1}\left(-\left((1-\eta)\bar{\mu}+\eta\right)\right) \\ &= \eta\frac{M}{2}(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-2\eta-1}\left(\bar{\mu}-1\right)-\frac{M}{2}(\eta-1)\bar{\mu}_j^{2\eta-2}\bar{\mu}^{-2\eta} \\ &= -\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}\left((1-\eta)\bar{\mu}+\eta\right)+\left((1-\eta)\bar{\mu}+\eta\right)\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-1}(\bar{\mu}-1)^{-1}\end{aligned}$$

$$\begin{aligned}C+E &= -\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}-\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}\left((1-\eta)\bar{\mu}+\eta\right)+\left((1-\eta)\bar{\mu}+\eta\right)\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-1}(\bar{\mu}-1)^{-1} \\ &= -\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}\left((1-\eta)\bar{\mu}+\eta+1\right)+\left((1-\eta)\bar{\mu}+\eta\right)\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-1}(\bar{\mu}-1)^{-1} \\ &= \bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}\left(-\eta\left((1-\eta)\bar{\mu}+\eta+1\right)+\left((1-\eta)\bar{\mu}+\eta\right)(\bar{\mu}-1)^{-1}\bar{\mu}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{\bar{\mu}^2}\frac{-C}{(C+E)^2} &= \frac{\eta\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-2}}{\left[\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta-1}\left(-\eta\left((1-\eta)\bar{\mu}+\eta+1\right)+\left((1-\eta)\bar{\mu}+\eta\right)(\bar{\mu}-1)^{-1}\bar{\mu}\right)\right]^2} \\ &= \eta\bar{\mu}_j^{1-\eta}\bar{\mu}^{\eta}(\bar{\mu}-1)^2\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]^{-2}\end{aligned}$$

Now

$$\begin{aligned}&\frac{d\left[\bar{\mu}_j^{1-\eta}\bar{\mu}^{\eta}(\bar{\mu}-1)^2\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]^{-2}\right]}{d\bar{\mu}} = \\ &(1-\eta)M(\bar{\mu}-1)^2\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]^{-2} \\ &+\eta\bar{\mu}^{\eta-1}\bar{\mu}_j^{1-\eta}(\bar{\mu}-1)^2\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]^{-2} \\ &+\bar{\mu}_j^{1-\eta}\bar{\mu}^{\eta}\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]^{-2}2(\bar{\mu}-1) \\ &-2\bar{\mu}_j^{1-\eta}\bar{\mu}^{\eta}(\bar{\mu}-1)^2\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]^{-3}\left(-2\bar{\mu}(1-\eta)^2+2\eta^2-\eta\right)\end{aligned}$$

This has the same sign as

$$\left[-(1-\eta)^2\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]\left\{(1-\eta)M(\bar{\mu}-1)+\eta\bar{\mu}^{\eta-1}\bar{\mu}_j^{1-\eta}(\bar{\mu}-1)+2\bar{\mu}_j^{1-\eta}\bar{\mu}^{\eta}\right\}$$
$$-2\bar{\mu}_j^{1-\eta}\bar{\mu}^{\eta}(\bar{\mu}-1)\left(-2\bar{\mu}(1-\eta)^2+2\eta^2-\eta\right)$$

Using again that

$$(1-\eta)M(\bar{\mu}-1)=2\left((1-\eta)\bar{\mu}+\eta\right)\bar{\mu}_j^{-\eta+1}\bar{\mu}^{\eta-1}$$

the above then has the same sign as

$$\left[-(1-2\eta+\eta^2)\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]\left\{2\left((1-\eta)\bar{\mu}+\eta\right)\bar{\mu}^{\eta-1}+(\eta+2)\bar{\mu}^{\eta}-\eta\bar{\mu}^{\eta-1}\right\}$$
$$-2\bar{\mu}^{\eta}\left(-2\bar{\mu}^2(1-2\eta+\eta^2)+\left(2\eta^2-\eta+2-4\eta+2\eta^2\right)\bar{\mu}-2\eta^2+\eta\right)=$$
$$\left[-(1-2\eta+\eta^2)\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]\left\{(\eta+2+2-2\eta)\bar{\mu}^{\eta}+\eta\bar{\mu}^{\eta-1}\right\}$$
$$-2\bar{\mu}^{\eta}\left(-2\bar{\mu}^2(1-2\eta+\eta^2)+\left(4\eta^2-5\eta+2\right)\bar{\mu}-2\eta^2+\eta\right)$$

This has the same sign as

$$\left[-(1-2\eta+\eta^2)\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]\left\{(\eta+2+2-2\eta)\bar{\mu}^{\eta}+\eta\bar{\mu}^{\eta-1}\right\}$$
$$-2\bar{\mu}^{\eta}\left(-2\bar{\mu}^2(1-2\eta+\eta^2)+\left(4\eta^2-5\eta+2\right)\bar{\mu}-2\eta^2+\eta\right)$$

which has the same sign as

$$\left[-(1-2\eta+\eta^2)\bar{\mu}^2+\left(2\eta^2-\eta\right)\bar{\mu}-\eta^2-\eta\right]\left\{(4-\eta)\bar{\mu}+\eta\right\}-2\bar{\mu}\left(-2\bar{\mu}^2(1-2\eta+\eta^2)+\left(4\eta^2-5\eta+2\right)\bar{\mu}-2\eta^2+\eta\right)$$
$$\bar{\mu}^3\left(\eta-2\eta^2+\eta^3\right)+\bar{\mu}^2\left(-3\eta^3+3\eta^2+5\eta-4\right)+\bar{\mu}\left(3\eta^3-6\eta\right)+\left(-\eta^3-\eta^2\right)$$

Let

$$G(\bar{\mu})=\bar{\mu}^3\left(\eta-2\eta^2+\eta^3\right)+\bar{\mu}^2\left(-3\eta^3+3\eta^2+5\eta-4\right)+\bar{\mu}\left(3\eta^3-6\eta\right)+\left(-\eta^3-\eta^2\right)$$

Then

$$G'''(\bar{\mu})=6\left(\eta-2\eta^2+\eta^3\right)>0$$

$$G''(\bar{\mu})=\bar{\mu}\left(6\eta^3-12\eta^2+6\eta\right)+\left(-6\eta^3+6\eta^2+10\eta-8\right)$$

Now

$$G''\left(\frac{\eta}{\eta-1}\right) = \frac{1}{\eta-1}\left(\eta\left(6\eta^3 - 12\eta^2 + 6\eta\right) + (\eta-1)\left(-6\eta^3 + 6\eta^2 + 10\eta - 8\right)\right)$$
$$= \frac{1}{\eta-1}(10\eta - 8)(\eta-1) > 0$$

So $G''(\bar{\mu}) > 0$ for $\bar{\mu} > \frac{\eta}{\eta-1}$

$$G'(\bar{\mu}) = 3\bar{\mu}^2\left(\eta - 2\eta^2 + \eta^3\right) + 2\bar{\mu}\left(-3\eta^3 + 3\eta^2 + 5\eta - 4\right) + \left(3\eta^3 - 6\eta\right)$$

$$G'\left(\frac{\eta}{\eta-1}\right) = \frac{1}{(\eta-1)^2}\left(3\eta^2\left(\eta - 2\eta^2 + \eta^3\right) + 2\left(\eta^2 - \eta\right)\left(-3\eta^3 + 3\eta^2 + 5\eta - 4\right) + \left(1 - 2\eta + \eta^2\right)\left(3\eta^3 - 6\eta\right)\right.$$
$$= \frac{2\eta}{(\eta-1)^2}\left((\eta-1)(2\eta-1)\right) > 0$$

So $G'(\bar{\mu}) > 0$ for $\bar{\mu} > \frac{\eta}{\eta-1}$

$$G\left(\frac{\eta}{\eta-1}\right) = \left(\frac{\eta}{\eta-1}\right)^3\left(\eta - 2\eta^2 + \eta^3\right) + \left(\frac{\eta}{\eta-1}\right)^2\left(-3\eta^3 + 3\eta^2 + 5\eta - 4\right) + \left(\frac{\eta}{\eta-1}\right)\left(3\eta^3 - 6\eta\right) + \left(-\eta^3\right.$$
$$= \left(\frac{\eta}{\eta-1}\right)^2 > 0$$

So $G(\bar{\mu}) > 0$ for $\bar{\mu} > \frac{\eta}{\eta-1}$. Hence

$$d\left(\frac{1}{\bar{\mu}^2}\frac{-C}{(C+E)^2}\right)/d\bar{\mu} > 0$$

Finally, I show that $d\bar{\mu}/dM > 0$ to establish that C, E, and $\frac{1}{\bar{\mu}^2}\frac{-C}{(C+E)^2}$ is increasing in M. From

$$((1-\eta)\bar{\mu} + \eta) + \bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2} = 0$$

$$\bar{\mu}_j = \left[M\bar{\mu}^{1-\eta} + (1-M)\mu_k^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

We have

$$((1-\eta)\bar{\mu}+\eta)\left(M\bar{\mu}^{1-\eta}+(1-M)\mu_k^{1-\eta}\right)+\bar{\mu}^{1-\eta}(\bar{\mu}-1)(\eta-1)\frac{M}{2}=0$$

$$(1-M)\mu_k^{1-\eta}\left((1-\eta)\bar{\mu}+\eta\right)+\bar{\mu}^{1-\eta}\left(M\left(\bar{\mu}-\eta\bar{\mu}+\eta\right)-\frac{M}{2}\left(\bar{\mu}-1-\bar{\mu}\eta+\eta\right)\right)=0$$

$$(1-M)\mu_k^{1-\eta}\left((1-\eta)\bar{\mu}+\eta\right)+\bar{\mu}^{1-\eta}\left((\bar{\mu}-\bar{\mu}\eta)\frac{M}{2}+\eta\frac{M}{2}+\frac{M}{2}\right)=0$$

$$(1-M)\mu_k^{1-\eta}\left((1-\eta)\bar{\mu}+\eta\right)+\frac{M}{2}\bar{\mu}^{1-\eta}\left(\bar{\mu}-\bar{\mu}\eta+\eta+1\right)=0$$

$$(1-M)\mu_k^{1-\eta}\eta+(1-\eta)(1-M)\mu_k^{1-\eta}\bar{\mu}+\frac{M}{2}\bar{\mu}^{1-\eta}\left(\eta+1\right)+\frac{M}{2}\bar{\mu}^{2-\eta}\left(1-\eta\right)=0$$

Let F denote the above equation. Applying the implicit function theorem:

$$\frac{\partial F}{\partial M}=-\mu_k^{1-\eta}\eta-(1-\eta)\mu_k^{1-\eta}\bar{\mu}+\frac{1}{2}\bar{\mu}^{1-\eta}\left(\eta+1\right)+\frac{1}{2}\bar{\mu}^{2-\eta}\left(1-\eta\right)$$

$$\frac{\partial F}{\partial \bar{\mu}}=(1-\eta)(1-M)\mu_k^{1-\eta}+(1-\eta)\frac{M}{2}\bar{\mu}^{-\eta}\left(\eta+1\right)+(2-\eta)\frac{M}{2}\bar{\mu}^{1-\eta}\left(1-\eta\right)$$

$$\begin{aligned}\frac{d\bar{\mu}}{dM}&=-\frac{-\mu_k^{1-\eta}\eta-(1-\eta)\mu_k^{1-\eta}\bar{\mu}+\frac{1}{2}\bar{\mu}^{1-\eta}\left(\eta+1\right)+\frac{1}{2}\bar{\mu}^{2-\eta}\left(1-\eta\right)}{(1-\eta)(1-M)\mu_k^{1-\eta}+(1-\eta)\frac{M}{2}\bar{\mu}^{-\eta}\left(\eta+1\right)+(2-\eta)\frac{M}{2}\bar{\mu}^{1-\eta}\left(1-\eta\right)}\\&=-\frac{-\mu_k^{1-\eta}\left(\eta+(1-\eta)\bar{\mu}\right)+\frac{1}{2}\bar{\mu}^{1-\eta}\left(\eta+\bar{\mu}\left(1-\eta\right)+1\right)}{(1-\eta)(1-M)\mu_k^{1-\eta}+(1-\eta)\bar{\mu}^{-\eta}\frac{M}{2}\left(\eta+(1-\eta)\bar{\mu}+\bar{\mu}+1\right)}\end{aligned}$$

I claim that

$$\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2}\leq 1 \tag{6}$$

This implies $\frac{d\bar{\mu}}{dM}>0$ if true. Now

$$\begin{aligned}&\frac{\partial\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2}}{\partial M}=\\&\frac{1}{2}\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\\&+\frac{d\bar{\mu}}{dM}\frac{M(\eta-1)}{2}\left(\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta}(\bar{\mu}(1-\eta)+\eta+\bar{\mu}-1)+\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\mu_j^{\eta-2}\mu_j^{\eta}\mu^{-\eta}M\right)=\\&\frac{1}{2}\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}+\frac{d\bar{\mu}}{dM}\frac{M(\eta-1)}{2}\bar{\mu}_j^{\eta-1}\bar{\mu}^{-\eta}\left(\bar{\mu}(1-\eta)+\eta+\bar{\mu}-1+\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\mu_j^{\eta-1}M\right)\end{aligned}$$

Since $((1-\eta)\bar{\mu}+\eta)+\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2}=0$, so the expression above is positive if $\frac{d\bar{\mu}}{dM}>0$.

Note that equation (6) is true for $M=1$, so $\frac{d\bar{\mu}}{dM}>0$ is true for $M=1$, and $\frac{\partial\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2}}{\partial M}>$

0 is true for $M = 1$. Starting from $M = 1$, a decrease in M decreases the LHS of (6), so equation (6) still holds. This implies that, for $M \in (0,1)$, $\bar{\mu}^{-\eta+1}(\bar{\mu}-1)(\eta-1)\bar{\mu}_j^{\eta-1}\frac{M}{2} \leq 1$, hence $\frac{d\bar{\mu}}{dM} > 0$.

TFP estimation

I assume the production function is given by

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \eta_{it}$$

where y_{it}, k_{it}, l_{it} are log output, log capital, and log labour respectively. ω_{it} is the productivity, and η_{it} is an error term. The firm is assume to know ω_{it} before making its investment decision. That is, k_{it} depends on ω_{it-1} which may correlate with ω_{it}, which gives rise to the simultaneity problem if we only regress y_{it} on k_{it} and l_{it}. Assume that l_{it} is variable and chosen in the current period.

The solution to the firm's problem gives an equation for investment:

$$i_{it} = i\left(\omega_{it}, k_{it}\right)$$

which can be inverted (under regularity conditions) to yield

$$\omega_{it} = h(i_{it}, k_{it})$$

Define

$$\phi_{it} = \beta_0 + \beta_k k_{it} + h\left(i_{it}, k_{it}\right)$$

From the production function, we obtain

$$y_{it} = \beta_l l_{it} + \phi_{it} + \eta_{it}$$

Now run a two stage regression. The first stage results in an estimate of $\hat{\beta}_l$. For the second stage, consider the expectation of $y_{it+1} - \hat{\beta}_l l_{it+1}$ conditional on time t information:

$$\begin{aligned} E_t\left(y_{it+1} - \hat{\beta}_l l_{it+1}\right) &= \beta_0 + \beta_k k_{it+1} + E_t\left(\omega_{it+1} | \omega_{it}, survival\right) \\ &= \beta_0 + \beta_k k_{it+1} + g\left(\omega_{it}, \hat{P}_t\right) \end{aligned}$$

where $\hat{P}_t$ is the probability of survival from t to $t+1$. This is estimated via a probit on a polynomial of investment and capital. $\hat{\beta}_k$ is then estimated via non-linear least squares, and productivity ω_{it} is then backed out from the production function.

I estimate firm productivity separately for each 2 digit SIC industry.

To estimate ϕ, I regress firms log TFP on aggregate log TFP, with indicator for whether the firm is large. ϕ is then the ratio of the estimate of large firm to the estimate of small firm, which is roughly 1.25. The regression results are given in the table below.

Dependent variable	Detrended log TFP_{it}	log TFP_{it}
$\hat{A}_t \times 1_{\{t<1985\}}$	0.9014	1.2409
	(0.0383)	(0.0799)
$\hat{A}_t \times 1_{\{i_t \in large_t\}} \times 1_{\{t<1985\}}$	0.1562	0.3188
	(0.0954)	(0.1980)
$\hat{A}_t \times 1_{\{t\geq 1985\}}$	1.4581	2.0161
	0.0544	(0.1146)
$\hat{A}_t \times 1_{\{i_t \in large_t\}} \times 1_{\{t\geq 1985\}}$	0.2011	0.6382
	(0.1351)	(0.2817)
Firm FE	No	Yes
N obs	206364	206364

Table 6: Firm TFP exposure to aggregate TFP